2011 Randolph Tournament 6th Grade Written Test Solutions

1. $2x = 5 \rightarrow x = 2.5$

2. Let P(hitting C) = p. Then P(hitting B) = 3p and P(hitting A) = 6p. The probability of one of the events occurring is 1, so $p + 3p + 6p + \frac{1}{21} = 1 \rightarrow p = \frac{2}{21}$. So P(hitting B) = $3 \cdot \frac{2}{21} = \frac{2}{7}$.

3. Since $0^1 = 0$, the product is 0.

4. Adding the two equations together gives $-13x - 13y = -39 \Rightarrow x + y = 3 \Rightarrow 7x + 7y = 21$.

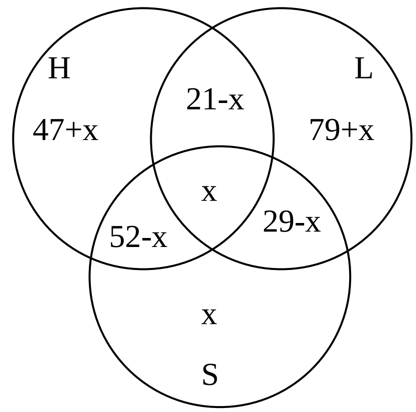
5. Make a Venn diagram. The sum of all the entries has to be 250.

$$47 + x + 21 - x + x + 52 - x + 79 + x + 29 - x + x = 250$$

$$x + 238 = 250$$

$$x = 12$$

The number of students taking Sanskrit and Lating but not Hebrew is $29 - x = 29 - 12 = 17$



6. The coordinate of E is 6. Since 8 is two to the right of 6, we look two to the right of B, which is C.

7. The number of positive factors is $(3+1)(2+1)(1+1) = 24$. That means there are also 24 negative factors, giving a total of 48.

8. $x \div \frac{1}{4} = x - 9 \rightarrow 4x = x - 9 \rightarrow 3x = -9 \rightarrow x = -3$.

9. $$\begin{array}{r} .6\overline{4} \\ 45\overline{)29.00} \\ \underline{27.0} \\ 200 \\ \underline{180} \\ 20 \end{array}$$

10. Sum of exterior angles of any polygon = $360°$. So number of sides = $360° \div 24° = 15$.

11. Possible birth orders given that at least three of the children are boys: BBBB, BBBG, BBGB, BGBB, GBBB. Since all five outcomes are equally likely, probability is $1/5$.

12. $2x + \frac{3}{12} + \frac{4}{16} = 10 \rightarrow 2x + \frac{1}{2} = 10 \rightarrow 2x = 9.5 \rightarrow x = 4.75$.

13. Either $x^2 = 9^2 + 12^2 \rightarrow x^2 = 225 \rightarrow x = 15$ or $x^2 + 9^2 = 12^2 \rightarrow x^2 = 63 \rightarrow x = \sqrt{63}$.

14. $\sqrt{36+64}+|10-12|=\sqrt{100}+|-2|=10+2=12$.

15. 400 = 20^2 and 1024 = 32^2, so there are $32-20+1=13$ perfect squares between the two.

16. P(red king) = $\frac{2}{52}$ and P(black card given that red king has been taken) = $\frac{26}{51}$.

So joint probability = $\frac{2}{52}\cdot\frac{26}{51}=\frac{1}{51}$.

17. To find *A*, consider numbers that are 3 more than consecutive multiples of 11 and find one that is two more than a multiple of 7: 3, 14, 25, 36, 47, **58**, 69, … . So *A* = 58. Similarly, *B* = 27.
So $A+B=58+27=85$.

18. If the first term of an arithmetic sequence is *a* and the common difference is *d* then the n^{th} term is given by $a+(n-1)d$. Thus $a+9\cdot3=74\Rightarrow a+27=74\Rightarrow a=47$.

19. $9009=3^2\cdot7\cdot11\cdot13$, so the largest prime factor is 13.

20. $3\uparrow\frac{1}{2}=3+\frac{1}{2}-3\div\frac{1}{2}=3.5-6=-2.5$.

21. The circumference of a circle is 2π times the radius. So $2\pi r=16\rightarrow r=\frac{16}{2\pi}=\frac{8}{\pi}$.

22. First, ignoring the restriction, each topping can either appear or not appear to make a given combination, so there are 2^6 = 64 total topping combinations. However, of there, 2^4 = 16 have both pickles and onions (since the other four toppings can either appear or not). The total is 64 – 16 = 48.

23. The only number that is neither prime nor composite is 1, so the probability of success is $\frac{5}{6}$. The odds of success are therefore $\frac{5}{6}:\frac{1}{6}=5:1$.

24. 255 = 2^8 – 1 = $100000000_2 - 1_2 = 11111111_2$.

25. The order of the genders has to be GBGBGBG (wrapped into a circle). There are 4! ways of arranging the girls and 3! ways of arranging the boys, so the total is $4!\cdot3!=24\cdot6=144$.

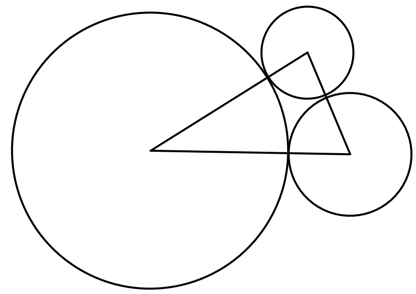
26. Using up a fraction *f* of the worms is equivalent to leaving (1 – *f*) of the worms. Thus if *x* is the number of worms Valerie had to start with, $\frac{3}{4}\cdot\frac{1}{2}\cdot\frac{1}{3}x=14\rightarrow\frac{1}{8}x=14\rightarrow x=112$.

27. $x=4+3\cdot(-x)\rightarrow x=4-3x\rightarrow4x=4\rightarrow x=1$.

28. 14 days = 2 weeks, while a dozen weeks = 12 weeks. Thus in the second scenario we have half as many cats working for six times as long. They can therefore catch three times as many, or 108, rats.

29. Let the value of a Maharba be m, the value of a Xela be x, and the value of an Ailuj be a. Then the relations described translate to $m = x + a$ and $2x = m + 2a$. Substituting the first into the second gives $2x = x + 3a \Rightarrow x = 3a$. Substituting this into the first equation gives $m = 4a$.

30. Subtracting x% off a price is the same as multiplying by (100 – x)%. So if the original price of the jacket was \$$P$, then $80\% \cdot 70\% \cdot 60\% \cdot P = 42 \rightarrow \frac{4}{5} \cdot \frac{7}{10} \cdot \frac{3}{5} P = 42 \rightarrow P = \frac{5}{4} \cdot \frac{10}{7} \cdot \frac{5}{3} \cdot 42 = 125$.



TB1

The sides of the triangle formed are 3 + 5, 3+12, and 5 + 12, or 8, 15, and 17. This is a Pythagorean triple, so the triangle is a right triangle. Its area is therefore. $0.5 \cdot 8 \cdot 15 = 60$.

TB2

Examining the units digits of successive powers of 3, 5, and 7, reveals that they repeat as follows:
3: 3, 9, 7, 1, 3, 9, 7, 1, …
5: 5, 5, 5, …
7: 7, 9, 3, 1, 7, 9, 3, 1, …
The exponents are all multiples of 4, so the corresponding units digit is 1 + 5 + 1 = 7.

TB3

The number of ways of arranging 6 letters is 6! = 720. Two pairs of letters (the B's and the E's) are indistinguishable, however, so we have to divide by 2! twice. The total number of arrangements is therefore $\frac{6!}{2! \cdot 2!} = \frac{720}{2 \cdot 2} = 180$.