2011 Randolph Tournament 5th Grade Written Test Solutions

1. The smaller area is ¼ of the clock area, so $\pi r^2 \div 4 = 36\pi \text{ in}^2 \div 4 = 9\pi \text{ in}^2$.

2. $3x - 17 = 22 \rightarrow 3x = 39 \rightarrow x = 13.$

3. $\frac{x}{y} = \frac{4}{7} \rightarrow x = \frac{4}{7}y$. If $y = 28$, then $x = \frac{4}{7} \cdot 28 = 16.$

4. Let x = # of pirates in the band. Then $x/3$ pirates have only 1 foot, while the rest have 2 feet. So $\frac{x}{3} + 2 \cdot \frac{2x}{3} = 55 \rightarrow \frac{5x}{3} = 55 \rightarrow x = 33.$ The number of pirates who have not lost a foot is $\frac{2}{3} \cdot 33 = 22.$

5. The sum of the angles in a triangle is 180°, so the sum of the two unknown angles is 102°. Let the smaller of the unknown angles be x°. Then $x + 2x = 102 \rightarrow 3x = 102 \rightarrow x = 34.$ So the smallest angle is 34°.

6. The prime numbers are 37, 41, 43, 47, 53, and 59. Their sum is 280.

7. Plugging in the four possibilities shown gives the following results: $5 + 4 \div 3 \times 2 - 1 = 20/3$, $5 - 4 + 3 \div 2 \times 1 = 5/2$, $5 - 4 + 3 \times 2 \div 1 = 7,$ and $5 + 4 - 3 \times 2 \div 1 = 3.$ So the answer is D.

8. $\frac{2}{5} + \frac{5}{3} - \frac{4}{7} = \frac{42}{105} + \frac{175}{105} - \frac{60}{105} = \frac{42 + 175 - 60}{105} = \frac{157}{105}.$

9. Area of a parallelogram = base × height = 20 × 8 = 160.

10. There are 5 choices for the first jelly bean, 4 for the second, etc., so the number of arrangements is $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5! = 120.$

11. The incorrect date was November 7. Now, counting ahead by 4 weeks, July 11 is the same day of the week as August 8, which is the same as September 5, which is the same as October 3, which is the same as October 31, which is the same (counting by one week) as November 7. So the original trip was also scheduled to be on a Monday.

12. $\frac{45 \text{ miles}}{1 \text{ hour}} = \frac{45 \cdot 5280 \text{ feet}}{1 \cdot 3600 \text{ seconds}} = \frac{66 \text{ feet}}{1 \text{ second}}.$

13. We need to choose 2 corners out of the 8 in the room. There are 8 choices for the first and 7 for the second, which is 56 choices, but this double counts every pair of corners, So the answer is $56/2 = 28.$

14. We know distance = rate × time, so letting t = the number of hours he rode, we have $100 = 16t \rightarrow t = \frac{100}{16} = \frac{25}{4} = 6\frac{1}{4}.$ So he finished $6\frac{1}{4}$ hours after he started; that is, at 2:45 p.m.

15. Each number in the pattern is double the previous one, so the next to numbers are 96 and 192. Their sum is 288.

16. $120\% \times 40\% \times 150\% = \frac{6}{5} \times \frac{2}{5} \times \frac{3}{2} = \frac{18}{25} = \frac{72}{100} = 0.72$

17. Since the median of the three numbers is 20, the middle number is 20. Since their average is 22, their sum is 3×22 = 66. That means the sum of the two unknown numbers is 66 – 20 = 46. Since the two other numbers have to be positive integers, the smallest that one of them could be is 1, which would make the other one 46 – 1 = 45.

18. $\sqrt{x+1} + 12 = 37 \rightarrow \sqrt{x+1} = 25 \rightarrow x + 1 = 25^2 = 625 \rightarrow x = 624.$

19. Let the radius of the smaller circle be 2*x*. Then the radius of the bigger circle is 3*x*. Their areas are, respectively, $4\pi x^2$ and $9\pi x^2$. The area of the portion of the larger circle that is *outside* the smaller circle is $9\pi x^2 - 4\pi x^2 = 5\pi x^2$. This represents $\frac{5\pi x^2}{9\pi x^2} = \frac{5}{9}$ of the larger circle.

20. $-(3-(-2-(-8))) - (-6) = -(3-6) - (-6) = -(-3) - (-6) = 3 + 6 = 9.$

21. Subtracting the second equation from the first gives $x + y = -10 \rightarrow |x + y| = |-10| = 10.$

22. $2 \downarrow 3 = 2^3 - 3^2 = 8 - 9 = -1.$

23. One of the first six non-negative integers is zero! So the product is 0.

24. The area that the goat can graze consists of ¾ of a circle of radius 50' plus ¼ of a circle of radius 10', which is the bit that he can graze beyond the corner of the 40' wall. The sum of the areas of these two pieces is $\frac{3}{4} \cdot 2500\pi \text{ ft}^2 + \frac{1}{4} \cdot 100\pi \text{ ft}^2 = (1875\pi + 25\pi) \text{ ft}^2 = 1900\pi \text{ ft}^2.$

25. At 2:30, the minute hand is pointing at the 6, while the hour hand is halfway between the 2 and the 3. There are 30° between each consecutive pair of numbers on a clock, so the total number of degrees is $3 \cdot 30° + 15° = 105°.$

26. $\sqrt{169} - \sqrt{144} = 13 - 12 = 1 = \sqrt{1}.$

27. Let *x* = # of cows a 235-acre farm can support. Then $\frac{x}{235} = \frac{2}{5} \rightarrow x = \frac{2}{5} \cdot 235 = 94.$

28. The factors of 24 are $\pm 1,\ \pm 2,\ \pm 3,\ \pm 4,\ \pm 6,\ \pm 8,\ \pm 12,$ and $\pm 24.$ There are 16 such factors.

29. Let *x* = # of pieces of wood in 215 seconds. Then $x = \frac{84}{1 \text{ minute}} \cdot 215 \text{ seconds} \cdot \frac{1 \text{ minute}}{60 \text{ seconds}} = 301.$

30. Let *x* = # of quarters. Then 42 – *x* = # of nickels. Since the values of the two types of coins are the same, we have $25x = 5(42 - x) \rightarrow 25x = 210 - 5x \rightarrow 30x = 210 \rightarrow x = 7 \rightarrow 42 - x = 35.$ So the difference between the number of nickels and quarters is 35 – 7 = 28.

TB1

The possible sums are 2, 3, …, 11, all of which are prime or composite, so the probability that the sum is prime or composite is 1.

TB2

Since the perimeter of the smaller square is 8, its side length is 2. Since the ratio of the side lengths is 1:3, the side length of the bigger square is 6. The shaded area is the difference between the areas of the squares, which is $6^2 - 2^2 = 36 - 4 = 32.$

TB3

Writing out the factorials as multiplications and reducing common terms gives

$$\frac{12!}{8!6!} = \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = \frac{12 \cdot 11 \cdot 10 \cdot 9}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = \frac{11 \cdot 3}{2 \cdot 1} = 16.5.$$