

# Grissom Math Tournament

# 20

# GEOMETRY

# 14



## Written Test

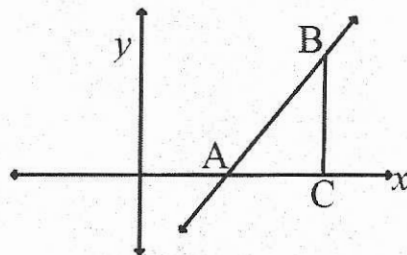
1. Sixty minutes will be allowed for completing this examination. The monitor will keep time. Students must stay in the room for the full sixty minutes.
2. No calculators, books, notes, or other aides may be used. Your monitor will supply scratch paper; you may not furnish your own. If you need more scratch paper during the test, raise your hand and the monitor will bring it to you.
3. You will receive four points for each correct answer minus one point for each incorrect answer on the twenty-five multiple choice questions. There are three tie breakers at the end of the test. Correct answers on the tie breakers are worth one-tenth of a point. Your score on the written test is the sum of these two scores.
4. If there are ties after the scores are computed as described in point 3 above, we will break them by counting number 25, then number 24, then number 23, and so on in this order as tie breakers.
5. Please give the monitor your answer sheet before you leave. You may keep the test copy. ***Be sure to bubble your student number in the appropriate place on your answer sheet. Otherwise, your paper will not be graded.***



1. The circumference of a circle is  $32\pi$ . Calculate the area of the circle in terms of  $\pi$ .  
 A.  $4\pi\sqrt{2}$     B.  $64\pi$     C.  $256\pi$     D.  $8\pi\sqrt{2}$     E. None of these

2. Given two points  $A(4,0)$  and  $C(15,0)$ , as well as the equation of  $\overleftrightarrow{AB}$ , which is  $y = 2x - 8$ , then what is the area of triangle ABC?

A. 100    B. 121    C. 144    D. 169    E. None of these



3. In triangle GHS,  $m\angle G = 5x - 15$ ,  $m\angle H = x + 23$ , and  $m\angle S = 4x + 2$ . Which description is accurate for triangle GHS?

A. acute scalene    B. obtuse scalene    C. acute isosceles    D. obtuse isosceles    E. None of these

4. The measure of an angle is  $33^\circ$  less than twice the complement of the angle. What is the supplement of this angle in degrees?

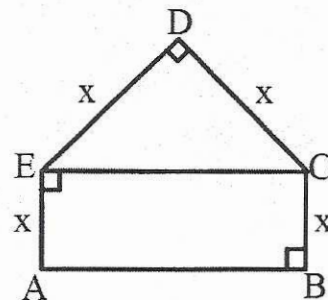
A. 71    B. 109    C. 49    D. 131    E. None of these

5. In a certain quadrilateral ABCD, the measures of angles A, B, C, and D are  $x$ ,  $2x$ ,  $3x$ , and  $4x$  respectively. What is the most precise name which can be assigned to this quadrilateral?

A. trapezoid    B. rectangle    C. kite    D. parallelogram    E. None of these

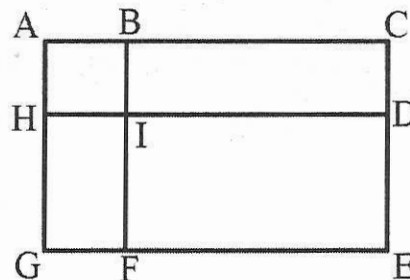
6. Isosceles right triangle CDE and rectangle ABCE have a common side  $\overline{EC}$ . If the perimeter of ABCDE is  $16 + 4\sqrt{2}$ , find the area of rectangle ABCE.

A.  $8\sqrt{2}$     B. 4    C. 16    D.  $16\sqrt{2}$     E. None of these



7. In rectangle ACEG,  $AH = \frac{1}{3}AG$  and  $AB = \frac{1}{3}BC$ . What is the ratio of the area of rectangle FIDE to the area of rectangle ABIH?

A. 2:1    B. 4:1    C. 6:1    D. 9:1    E. None of these



8. The sum of the interior angles of a regular polygon is  $1260^\circ$ . How many total diagonals can be drawn in this polygon?

A. 14    B. 27    C. 36    D. 9    E. None of these

9. Pretend you have a 24-hour analog wristwatch (instead of displaying numbers from 1 to 12, it displays numbers from 1 to 24 where the hour hand is between the 12 and 24 during the p.m. hours). The minute hand still makes one entire rotation every hour, but the hour hand completes one rotation per day. You wake up at 2:15 a.m. and look at your watch. What is the measure of the smaller angle formed by the hour and minute hands?

- A.  $\frac{45}{2}$       B.  $\frac{75}{4}$       C.  $\frac{225}{4}$       D.  $\frac{495}{4}$       E. None of these

10. A sphere is inscribed in a right circular cylinder. What is the ratio of the surface area of a sphere to the lateral area of the cylinder?

- A. 2:3      B. 1:1      C.  $\pi:1$       D. 2:3      E. None of these

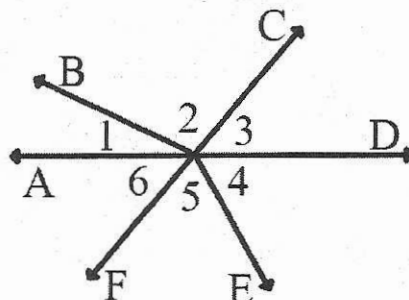
11. Which of the following are ways to prove that two triangles are congruent?

- I. ASA      II. SSA      III. AAA      IV. SAS

- A. I, IV only      B. I, II, IV only      C. III only      D. II, IV only      E. None of these

12. In the given diagram,  $\overline{FC}$  and  $\overline{AD}$  intersect at P. If  $m\angle 1 = x^2 + 1$ ,  $m\angle 2 = 4x + 7$ ,  $m\angle 4 = 6x + 4$ , and  $m\angle 5 = 3x + 10$ , find  $m\angle 3$  in degrees.

- A. 6      B. 74      C. 12      D. 106      E. None of these

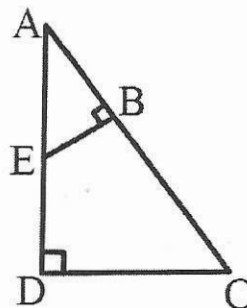


13. The length, width, and height of a rectangular solid are in the ratio of 3:4:12. If the space diagonal of the prism is 26 inches, what is the surface area of this solid in square inches?

- A. 432      B. 1152      C. 768      D. 384      E. None of these

14. In the diagram with right triangles ABE and ADC, E is the midpoint of  $\overline{AD}$ . If  $DC = 6$ , and  $AC = 10$ , what is the length of  $\overline{AB}$ ?

- A.  $\frac{16}{5}$       B. 3      C. 4  
D.  $\frac{12}{5}$       E. None of these



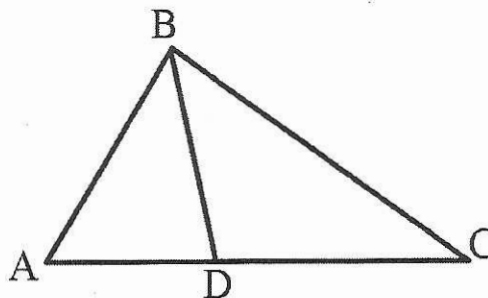
15. The center of a circle with radius 6 is located at the point (1, 2), and point P is located at (13, 7). If  $a$  is the distance from point P to the closest point on the circle, and  $b$  is the distance from point P to furthest point on the circle, then find  $a + b$ .

- A. 13      B. 7      C. 19      D. 26      E. None of these

16. In the given diagram,  $\overline{BD}$  bisects  $\angle ABC$ .

If  $AB = 6$ ,  $AD = 3$ , and  $BC = 4$ , what is  $AC$ ?

- A. 2                      B. 5                      C.  $\sqrt{52}$   
D. 8                      E. None of these



17. A tire rolls 110 yards in 42 revolutions. Using  $\frac{22}{7}$  as an approximation for  $\pi$ , find the diameter of the tire **in inches**.

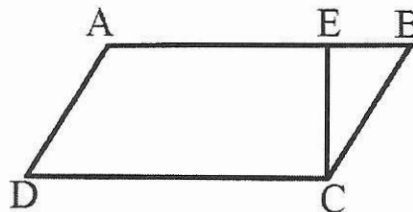
- A.  $5/6$                       B. 24                      C.  $2/3$                       D. 30                      E. None of these

18. The sides of a triangle are 10, 17, and 21 centimeters. What is the length of the altitude (in centimeters) drawn to the longest side?

- A. 8                      B.  $\frac{170}{21}$                       C. 6                      D. 9                      E. None of these

19. Find the area of parallelogram ABCD, given that  $\overline{CE} \perp \overline{AB}$ ,  $AE = 7$ ,  $CD = 10$ , and  $BC = 5$ .

- A. 50                      B. 20                      C. 28  
D. 35                      E. None of these

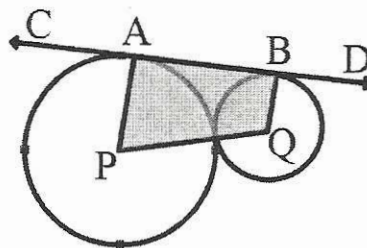


20. An isosceles trapezoid with bases of length 12 and 16 is inscribed in a circle of radius 10. The center of the circle lies in the interior of the trapezoid. What is the area of this trapezoid?

- A. 14                      B. 196                      C. 98                      D. 392                      E. None of these

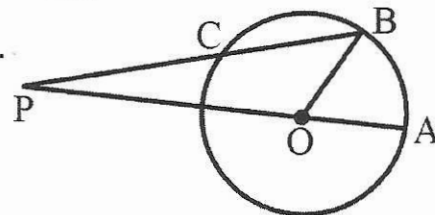
21. In the figure, line CD is tangent to circles P and Q at points A and B, respectively. If  $PA = 16$  and  $QB = 4$ , find the area of the shaded region.

- A. 64    B. 144    C. 160    D. 256    E. None of these



22. In the figure,  $\overline{PB}$  and  $\overline{PA}$  are secants,  $PC = OA$ , and  $m\angle BOA = 60^\circ$ . What is the measure of  $\angle BPA$ ?

- A. 10    B. 15    C. 20    D. 30    E. None of these



23. In a certain rhombus, the longest diagonal length is  $16\sqrt{3}$  units and its perimeter is 64. What is the area of the rhombus?

- A. 64                      B.  $64\sqrt{3}$                       C.  $128\sqrt{3}$                       D.  $96\sqrt{3}$                       E. None of these

24. Three vertices are chosen randomly from regular octagon MYOCTAGN. What is the probability that the triangle formed using these three vertices is NOT a right triangle?

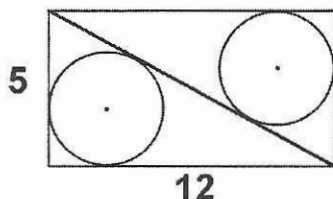
- A.  $\frac{3}{5}$       B.  $\frac{4}{7}$       C.  $\frac{6}{11}$       D.  $\frac{7}{13}$       E. None of these

25. In triangle ABC, D lies on  $\overline{BC}$ , E lies on  $\overline{AC}$ , and F lies on  $\overline{AB}$  such that  $\overline{AD}$ ,  $\overline{BE}$ , and  $\overline{CF}$  are concurrent at point P. If  $\frac{BD}{DC} = \frac{4}{3}$  and  $\frac{EC}{AC} = \frac{2}{5}$  then what is the value of  $\frac{FP}{PC}$ ?

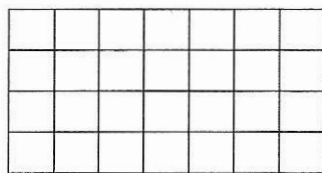
- A.  $\frac{8}{9}$       B.  $\frac{9}{8}$       C.  $\frac{12}{17}$       D.  $\frac{12}{29}$       E. None of these

### TIE BREAKERS:

TB1: The rectangle below has two circles inside which are each tangent to two sides of the rectangle as well as the rectangle's diagonal. What is the distance between the centers of the two circles?



TB2: How many rectangles of any size can be formed from the given 4 by 7 grid of squares (without rearranging the squares)?



TB3: How many non-congruent triangles have integer side lengths and a perimeter less than 10 units?