2014 Hoover High School Math Tournament: Geometry Written Test

1. Find the circumference of a circle in which a 50° sector has an area of 15π.

A. 108π B. $6\sqrt{3}\pi$ C. $3\sqrt{6}\pi$ D. $12\sqrt{3}\pi$ E. NOTA

2. What is the total number of diagonals in a regular hectogon?

A. 1175   B. 3975   C. 4850   D. 4950   E. NOTA

3. Hoth is at the point (3, 5) on the galactic maps (a normal coordinate grid). Dagobah is at (21, 21). Luke needs to get from Hoth to Dagobah but has to meet a space trader at some point along the Corellian Trade Spine. The Corellian Trade Spine is along the line Y=1. What is the total length of the shortest route Luke can travel to the trader then to Dagobah, assuming that he takes straight paths… no hyperjumps.

A. 20   B. $\sqrt{145}$ C. 30   D. $\sqrt{195}$ E. NOTA

4. A regular tetrahedron has a surface area of $\frac{25\sqrt{3}}{4}$. What is the height of this solid?

A. $\frac{5\sqrt{6}}{6}$ B. $\frac{125}{4}$ C. $\frac{25\sqrt{2}}{8}$ D. $\frac{25}{12}$ E. NOTA

5. Which special concurrent point in a triangle is not always on Euler's segment?

A. Incenter   B. Circumcenter   C. Centroid   D. Orthocenter   E. NOTA

6. Evan stores Poké-balls in a special edition Poké-box in the shape of a rectangular prism. The dimensions of this box are 6cm, 18cm, and 18cm. A Poké-ball has a diameter of 6cm. Assuming Evan cannot dissect a Poké-ball, how much empty space is there in the box when the box is at maximum Poké-ball capacity?

A. $216 - 144\pi$ B. $648 - 216\pi$ C. $1944 - 324\pi$ D. $5832 - 486\pi$ E. NOTA

7. A triangle is made from uniform weight cardboard and placed on a coordinate plane so that the vertices are on (2,-3), (4,6), and (-5,-7). What are the coordinates of the center of gravity of the triangle?

A. (-1,-1)   B. $\left(\frac{1}{3}, -\frac{4}{3}\right)$ C. $\left(\frac{9}{2}, \frac{13}{9}\right)$ D. $\left(\frac{4}{3}, \frac{13}{9}\right)$ E. NOTA

8. In the *xy*-coordinate plane, consider the "L" shaped figure bounded by horizontal and vertical segments with consecutive vertices (0, 0), (0, 5), (3, 5), (3, 1), (6, 1) and (6, 0). What is the slope of the line through the origin that divides the area of this figure in half?

A. $\frac{5}{7}$ B. $\frac{2}{3}$ C. $\frac{5}{4}$ D. $\frac{4}{3}$ E. NOTA

9. The sides of a triangle are 12, 12, and 16. What is the sum of the lengths of all three of its altitudes?

A. $\frac{16\sqrt{5}}{3}$ B. $\frac{22\sqrt{5}}{3}$ C. $\frac{32\sqrt{5}}{3}$ D. $\frac{44\sqrt{5}}{3}$ E. NOTA

10. $\overline{AB}$ is a chord in the great circle of a sphere. $\overline{CD}$ is another chord in the great circle that intersects $\overline{AB}$ at point E perpendicularly. $CE = 2$, $AE = 4$, and $BE = 3$. What is the surface area of the sphere?

A. 65π B. 49π C. 196π D. 36π E. NOTA

11. WXYZ is a rectangle with WX = 80 and XY = 24. V is a point on line segment WX such that XV > WV and ΔWVZ is similar to ΔXYV. Find WV.

A. 6   B. 8   C. 10   D. 12   E. NOTA

12. The frame of a hockey net is 6 feet wide and 4 feet high. A player, standing 30 feet away from and directly in front, centered up on the net, shoots a puck and it hits the very upper right corner of the frame. How far did the puck travel?

A. $6\sqrt{26}$ B. $2\sqrt{229}$ C. $5\sqrt{37}$ D. $\sqrt{952}$ E. NOTA

13. A large rectangle is partitioned into four rectangles by two line segments parallel to its sides. The areas of three of the resulting rectangles are shown. What is the area of the fourth rectangle?
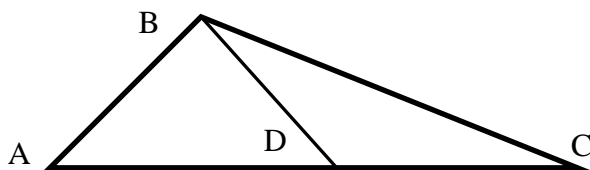
8	14
	22

A. 12   B. $\frac{25}{2}$ C. 10   D. $\frac{88}{7}$ E. NOTA

14. A circle with a 16 cm diameter has a sector whose area is 4π. What is the perimeter of this sector?

A. $16 + \pi$ B. $32 + 4\pi$ C. 64π D. 64π E. NOTA

15. In triangle ABC, A, C, and D are collinear, AB=BD=5, BC=7, and AC=9. What is AD/DC?


A. 8/3   B. 19/3   C. 19/8   D. 24/9   E. NOTA

16. Aneesh is stranded on an island with nothing to drink but spherical coconuts. The spherical coconuts have an outside surface area of 49π inches squared and have a thick layer of white husk of 1.5 inches thick. The rest of the coconut is 2/3 filled with water. How many whole coconuts must Aneesh drink to be sure he gets the recommended daily serving of 81π inches cubed of water?

A. 7   B. 9   C. 10   D. 12   E. NOTA

17. Solve for x:
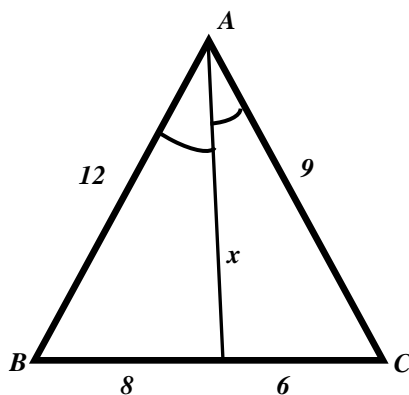
A. $\sqrt{65}$ B. $\frac{\sqrt{543}}{3}$ C. $2\sqrt{15}$ D. $\frac{10\sqrt{6}}{3}$ E. NOTA

18. Nathan believes that he can survive by only eating a genetically engineered fruit called hexafruit. It is in the shape of a regular hexagonal prism. The fruit has a height of 6 and has a base whose perimeter is 24. The hexafruit has a single, spherical shaped seed with a surface area of 9π inside and at the center of the fruit. The seed and any fruit within 0.5 from the seed is poisonous. How much volume of fruit can Nathan eat from each hexafruit without getting poisoned?

A. $72\sqrt{3}-\frac{32}{3}\pi$ B. $144\sqrt{3}-\frac{32}{3}\pi$ C. $72\sqrt{3}-\frac{28}{3}\pi$ D. $144\sqrt{3}-\frac{28}{3}\pi$ E. NOTA

19. The shortest diagonal of a regular hexagon is 8 cm in length. What is the area of the hexagon?

A. $96\sqrt{3}$ B. $32\sqrt{3}$ C. $384\sqrt{3}$.   D. $128\sqrt{3}$ E. NOTA

20. An open top rectangular box is used as a pencil box. Its dimensions are 10 cm, 10 cm, and 12 cm. A pencil is completely inside the box. What is the longest possible length for this pencil?

A. $10\sqrt{2}$ B. $2\sqrt{86}$ C. 32   D. $4\sqrt{61}$ E. NOTA

21. A right triangle is inserted into a cylinder so that one leg is the diameter of the base and the other leg is the height of the cylinder. If the radius of the cylinder is 7 cm and the area of the triangle is $49\sqrt{3}$, what is the lateral surface area of the cylinder?

A. $343\pi\sqrt{3}$ B. $42\pi\sqrt{3}$ C. $28\pi\sqrt{3}$ D. $98\pi\sqrt{3}$ E. NOTA

22. Let coplanar circles ⊙P and ⊙L represent the bases of two overlapping right cylinders with heights 5 and 7, respectively. Points X and Y are where the circles intersect. If each circle has a radius of 10 and XY=$10\sqrt{2}$, what is the volume of the resulting solid formed by the cylinders?

A. $750\pi+500$ B. $1700\pi+1000$ C. $950\pi+500$ D. $750\pi+1000$ E. NOTA

23. The Star Fleet emblem can be made from of a quadrilateral TREK inscribed in ⊙E as shown. $m\angle REK=120°$ and the area of ⊙E is 36π. If TREK were rotated 180° about $\overline{TE}$, what is the surface area of resulting solid?
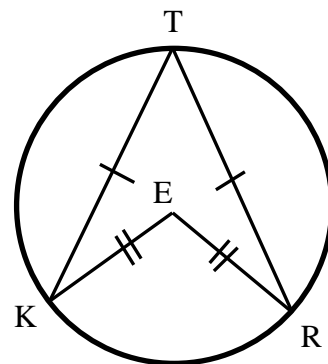
A. $81\pi + 18\pi\sqrt{3}$ B. $54\pi + 18\pi\sqrt{3}$ C. $81\pi + 21\pi\sqrt{3}$ D. $54\pi + 21\pi\sqrt{3}$ E. NOTA

24. Two gears, of diameters 24 and 36, are connected by a belt. The distance between the centers of the gears is 60. The distance from the center of the smaller gear to the point where the belt crosses itself is 24. What is the length of the belt?
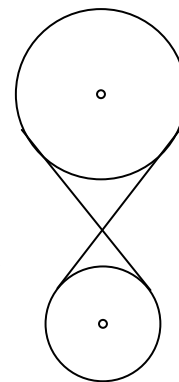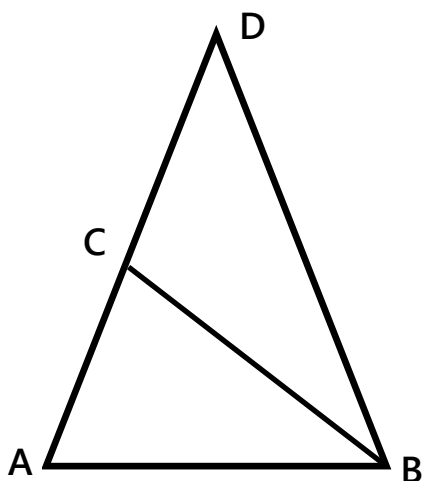
A. $54\pi + 60\sqrt{3}$ B. $40\pi + 60\sqrt{3}$ C. $54\pi + 72\sqrt{3}$ D. $40\pi + 30\sqrt{3}$ E. NOTA

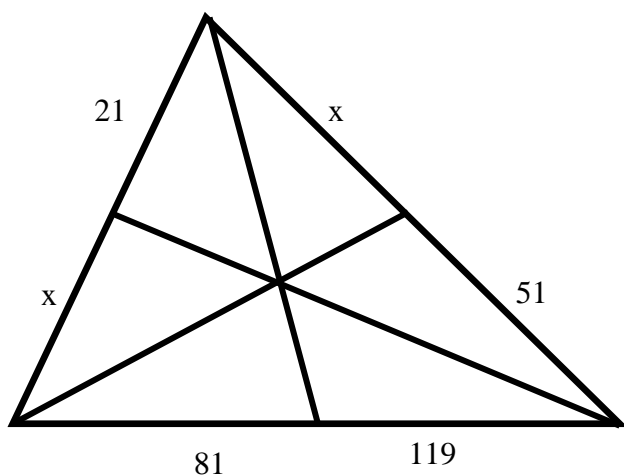25. Assuming that the chords pass through the intersection points of the two circles, solve for x:
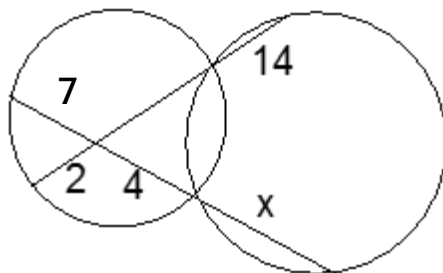
A. 14   B. 28   C. 94   D. 98   E. NOTA

TB1: In the following diagram, AB = BC = CD and AD = BD. Find the measure of angle D.

TB2: Solve for the length x below:

TB3: Given two externally tangent circles with radii a = 8 & b = 6. Find c, the radius of the smallest circle tangent to the other two circles and the common external tangent of the other two circles.