

Grissom Math Tournament

20 COMPREHENSIVE 14



Written Test

1. Sixty minutes will be allowed for completing this examination. The monitor will keep time. Students must stay in the room for the full sixty minutes.
2. No calculators, books, notes, or other aides may be used. Your monitor will supply scratch paper; you may not furnish your own. If you need more scratch paper during the test, raise your hand and the monitor will bring it to you.
3. You will receive four points for each correct answer minus one point for each incorrect answer on the twenty-five multiple choice questions. There are three tie breakers at the end of the test. Correct answers on the tie breakers are worth one-tenth of a point. Your score on the written test is the sum of these two scores.
4. If there are ties after the scores are computed as described in point 3 above, we will break them by counting number 25, then number 24, then number 23, and so on in this order as tie breakers.
5. Please give the monitor your answer sheet before you leave. You may keep the test copy. ***Be sure to bubble your student number in the appropriate place on your answer sheet. Otherwise, your paper will not be graded.***

1. Evaluate: $8 - 3[7 - 2(a^2 - b^2)]$ if $a = 4$ and $b = -3$.
A. -307 B. 29 C. 137 D. 175 E. 625
2. Simplify: $\frac{3}{3+i} + \frac{5}{2-4i}$
A. $\frac{7+14i}{10}$ B. $\frac{14+7i}{10}$ C. $\frac{19+17i}{10}$ D. $\frac{17+19i}{20}$ E. $\frac{19+17i}{20}$
3. Find the sum of the cubes of the roots of $x^2 - 6x + 4 = 0$.
A. 65 B. 72 C. 96 D. 120 E. 144
4. Find the value of $x+y$: $\begin{bmatrix} 5 & -4 \\ 1 & 9 \end{bmatrix} \begin{bmatrix} 7 & x & 3 \\ 1 & 9 & y \end{bmatrix} = \begin{bmatrix} 31 & -1 & 7 \end{bmatrix}$
A. 6 B. 7 C. 8 D. 9 E. 10
5. Evaluate: $(4^{-1} + 5^{-1})^{-2}$.
A. $\frac{20}{9}$ B. $\frac{40}{9}$ C. $\frac{400}{81}$ D. 41 E. 400
6. Solve for n , where $n! = 6!(7!)$.
A. 8 B. 9 C. 10 D. 12 E. 13
7. What is the third term in the harmonic sequence with $a_1 = \frac{2}{3}$ and $a_2 = \frac{4}{11}$?
A. $\frac{1}{16}$ B. $\frac{1}{4}$ C. $\frac{6}{19}$ D. $\frac{19}{6}$ E. 4
8. Find the sum of all integers between 1 and 101 that are perfect squares or perfect cubes but not both.
A. 255 B. 264 C. 320 D. 328 E. 355
9. Given triangle ABC with area 24, if D is the midpoint of \overline{AB} , E is the midpoint of \overline{BC} and F is the midpoint of \overline{AC} , find the area of trapezoid DBCF.
A. 6 B. 8 C. 12 D. 18 E. 20
10. If $\log_5 2 = x$, and $\log_5 3 = y$, then which of the following is equivalent to $\log_5 4320$?
A. $5x^5y^3$ B. $5x^3y^5$ C. $5 + 3x + 5y$ D. $1 + 5x + 3y$ E. $1 + 3x + 5y$

11. The Rocket City has decided to award \$2014 to the top scorers on the AMC-12 in 2014. The amounts of the awards will be \$1, \$7, \$49, and \$343. What is the least number of awards that can be given if the total value of the awards is exactly \$2014?
- A. 16 B. 26 C. 31 D. 41 E. 287
12. Point C(5, -2) is the midpoint of the line segment with endpoints A(8, -6) and B(x, y). Find the equation of the line perpendicular to line segment \overline{AB} and passing through point B.
- A. $3x - 4y = 2$ B. $3x - 4y = -2$ C. $4x + 3y = 12$ D. $4x + 3y = 14$ E. none of these
13. In the sequence $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots, \frac{1}{99}, \frac{1}{100}$, what percentage of the terms have non-repeating decimal expansions?
- A. 9% B. 15% C. 18% D. 21% E. 85%
14. Find the area of the region enclosed by the graph of the parametric equations: $x(t) = 6\cos t + 1$ and $y(t) = 8\sin t - 3$.
- A. 12π B. 16π C. 24π D. 48π E. 100π
15. The function $y = 2\log_3(-2x^2 + 7x - 3) - 4$ has y-intercept (0, b). If n is the number of integers in the domain of the function, find bn.
- A. -4 B. -2 C. 2 D. 4 E. 6
16. A cone is generated by rotating the region bounded by $2x - y = 12$, the y-axis and the x-axis about the x axis. Find the volume of this cone.
- A. 144π B. 288π C. 576π D. 864π E. 1728π
17. The distance between the points with polar coordinates $\left(6, \frac{3\pi}{2}\right)$ and $\left(8, \frac{7\pi}{4}\right)$ is in which of the following intervals?
- A. [5.5, 6) B. [6, 7) C. [7, 8) D. [8, 8.5) E. [8.5, 9)
18. The set of all points P(x, y) such that the difference between their distances to points S(-2, 0) and T(8, 0) is 6 satisfies the equation $Ax^2 - Cy^2 + Dx + Ey + F = 0$. Find the product of the x-coordinates of the points of intersection of this graph with the line $y=3$.
- A. $-\frac{81}{16}$ B. $-\frac{25}{4}$ C. $-\frac{9}{16}$ D. $\frac{9}{16}$ E. $\frac{25}{4}$
19. Find the sum of the roots of $\cos(3x) + \sin^2(3x) = 1$, where $0 \leq x \leq 2\pi$.
- A. 2π B. 4π C. 7π D. 8π E. 10π

20. What is the largest integer, k , where $\frac{2014!}{2014^k}$ is an integer?

- A. 1 B. 4 C. 39 D. 56 E. none of these

21. Evaluate $\lim_{n \rightarrow \infty} \left(1 + \frac{3}{n}\right)^{\frac{n}{2}}$

- A. $e^{\frac{1}{6}}$ B. $e^{\frac{2}{3}}$ C. e D. $e^{\frac{3}{2}}$ E. e^6

22. The solution set of $|x - |x - 4|| < 2$ is which of the following?

- A. $1 < x < 3$ B. $4 < x < 6$ C. $x < 4$ or $x > 6$ D. $x < 1$ or $x > 3$ E. none of these

23. In a rectangular coordinate system, a tangent from point $P(24, 7)$ to the circle $x^2 + y^2 = 400$ has a point of tangency $T(a, b)$ with $b > 0$. Find $a + b$.

- A. -4 B. -3 C. 11 D. 21 E. 28

24. If $\sum_{k=1}^{2014} 2^k + \sum_{k=1}^{2013} 2^k = 2^{(A+2014)} + 2^{(B+2014)} - 2^{(C+2014)}$, then find the value of $A + BC$.

- A. 0 B. 1 C. 2 D. 3 E. 4

25. Many 4 digit numbers can be written using the digits 0, 1, 2, 3, and 4. When these numerals are arranged in a sequence from least to greatest (for example, 1000, 1001, 1002, etc.) which term will be 2014?

- A. 135 B. 259 C. 359 D. 510 E. 634

Tie Breaker 1: Find the sum of all positive integral divisors of $49^{\log_{\sqrt{7}} 4}$.

Tie Breaker 2: How many values (in base 10) from 1 to 100 (inclusive) have the same number of digits in base 10 as they have when expressed in base 7?

Tie Breaker 3: What positive integer less than 1000 has the largest number of positive integral divisors?