2014 Hoover High School Mathematics Tournament
Comprehensive Written Test

1. If $a \oplus b = a^2 - b$ and $f(x) = 2x^2 - 3x + 2$, find the value of $f(3 \oplus 5)$.
A) 11 B) 14 C) 16 D) 22 E) NOTA

2. Abhay, Ishant, and Hemin can cut yards at rates of 40 minutes, 60 minutes, and 30 minutes per yard, respectively. Ishant begins cutting a yard, working by himself for 6 minutes before Abhay and Hemin join him to finish the yard. How long, in minutes, does it take from the time Ishant begins cutting the yard until the time the job is done?
A) 12 B) 14 C) 16 D) 18 E) NOTA

3. Diego, Yuan, Marshall, and Sunny are wrapping presents for the annual math team Secret Birthday Bash. All four wrap at constant rates and can wrap a total of 40 presents altogether in half an hour. If Andrew joins them, also wrapping at the same constant rate, then they can wrap 100 presents in what amount of time?
A) 40 minutes B) 45 minutes C) 60 minutes D) 75 minutes E) NOTA

4. A fly flies through the frame of an octahedron suspended from the ceiling. If a path is defined as entering through the interior of one of the octahedron's faces and passing through the interior of a different face, then how many different paths can the fly take?
A) 28 B) 36 C) 56 D) 72 E) NOTA

5. If $x + x^{-1} = 3$, find the value of $x^6 + x^{-6}$.
A) 322 B) 454 C) 656 D) 729 E) NOTA

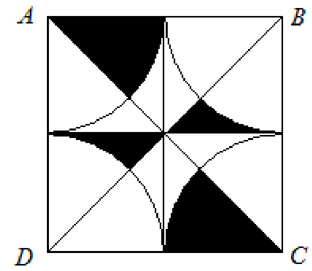
6. Find the positive difference between the lengths of the radii of the inscribed and circumscribed circles of a triangle with sides of lengths 48, 90, and 102.
A) 24 B) 33 C) 39 D) 42 E) NOTA

7. At 2:15 pm, how many degrees has an analog clock's hour hand passed since 9:40 am on the same day?
A) 122.5° B) 130° C) 137.5° D) 145° E) NOTA

8. Janice's coin jar contains \$7 each in pennies, nickels, dimes, and quarters; no other coins are in the jar. If she randomly chooses one coin from the jar, what is the probability that it is a nickel?
A) $\frac{10}{67}$ B) $\frac{5}{67}$ C) $\frac{2}{67}$ C) $\frac{8}{67}$ E) NOTA

9. Seven consecutive odd integers have a sum that is a perfect square. If the seven consecutive odd integers are all less than 1000, find their greatest possible sum.
A) 5041 B) 5329 C) 5625 D) 5929 E) NOTA

10. The diagram to the right consists of square $ABCD$ with side length $8\sqrt{3}$, the two diagonals of that square, the two line segments connecting the midpoints of opposite sides of that square, and four quarter-circles. The circles from which the quarter-circles are taken have centers at the vertices of square $ABCD$. Find the area of the shaded region.



A) 12π B) 48 C) $48-12\pi$ D) 24
E) NOTA

11. A David sequence is a finite, increasing sequence of integers where the first term is 1, every integer from 1 to n is used, and either the odd integers are repeated once or the even integers are repeated once (for example, 1,1,2,3,3,4,5,5,6 and 1,2,2,3,4,4 are both David sequences). If the sum of the terms of a David sequence is 374, find the greatest value of the final term of the David sequence.

A) 21 B) 22 C) 23 D) 24 E) NOTA

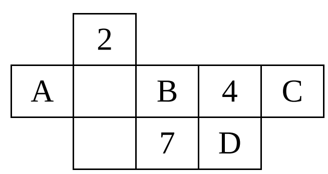
12. The plan is to give Richard and Danny ice cream every day for a five-day school week. However, for each day, there is a $\frac{1}{3}$ chance that the ice cream won't be delivered. What is the probability that Richard and Danny get their ice cream for at least two of the five days?

A) $\frac{47}{81}$ B) $\frac{151}{243}$ C) $\frac{74}{81}$ D) $\frac{232}{243}$ E) NOTA

13. A group consists of 60 students. Of those students, 36 like math and 32 like science. If m is the minimum number of students in this group who could like both subjects and M is the maximum number of students in this group who could like both subjects, find $M-m$.

A) 24 B) 16 C) 8 D) 0 E) NOTA

14. The integers from 1 through 9, inclusive, are to be Placed in the diagram to the right, one integer per square; the integers 2, 4, and 7 are already placed. Additionally, no two consecutive integers may be placed in any two squares that touch in any way, including at a corner. In which lettered square must the 6 be placed?



A) A B) B C) C D) D E) NOTA

15. In how many six-digit numbers do all six digits have the same parity (odd or even)? 0 may not be the leading digit of a six-digit number.

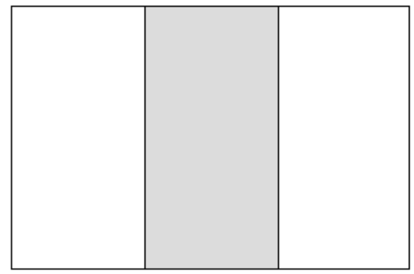
A) 15,625 B) 28,125 C) 21,875 D) 18,750 E) NOTA

16. Yilan and Danae have several magazines. If Yilan gave Danae four magazines, they have the same number of magazines. If Danae gave Yilan four magazines, then Yilan would have three times as many magazines as Danae. How many magazines does Yilan have?

A) 8 B) 12 C) 16 D) 20 E) NOTA

17. Xianming brought a bunch of Pixy Stix$_{TM}$ to math team practice. If all of the members had shown up and had taken three Pixy Stix$_{TM}$ each, there would have been 34 Pixy Stix$_{TM}$ left over. Instead, Jeremy and Eric didn't show up, and the members who did show up took four Pixy Stix$_{TM}$ each, leaving four Pixy Stix$_{TM}$ remaining. How many members does the math team have?

A) 38 B) 74 C) 76 D) 148 E) NOTA



18. Two squares, each with dimensions 30×30, are overlapped as in the diagram to the right, forming a large rectangle with dimensions 30×46. Find the area of the shaded region.

A) 360 B) 420 C) 480 D) 540 E) NOTA

19. Given the following statements:

1) No bad movies are given Oscar consideration.
2) Some bad movies have cult followings.

Which of the following is a logical conclusion?

A) No movie with a cult following is given Oscar consideration.
B) Movies without a cult following are not given Oscar consideration.
C) Some movies with cult followings are not given Oscar consideration.
D) There is some movie given Oscar consideration that is a bad movie.
E) NOTA

20. Find the tens' digit of the quantity 51^{2014}.

A) 0 B) 1 C) 3 D) 7 E) NOTA

21. One root of $f(x)=x^4-4x^3-5x^2-64x-336$ is $x=-4i$. If a and b are the real roots of $f(x)$, find the value of $\frac{a^2+b^2}{2ab}$.

A) 7 B) 16 C) $\frac{64}{21}$ D) $-\frac{64}{21}$ E) NOTA

22. If f_n represents the nth Fibonacci number, where $f_1=f_2=1$, then $\frac{f_n}{f_{n-1}}-\frac{1}{\left(\frac{f_{n-1}}{f_{n-2}}\right)}=1$ for $n\geq3$. Because of this relationship, find the value of $\lim_{n\to\infty}\frac{f_n}{f_{n-1}}$.

A) 2 B) $\frac{1+\sqrt{5}}{2}$ C) 1 D) $\frac{-1+\sqrt{5}}{2}$ E) NOTA

23. For a particular card game, the face cards are removed from a standard, 52-card deck, at which point three cards are deal from the remaining deck. What is the probability that a flush (all three cards of the same suit) is dealt?

A) $\frac{12}{247}$ B) $\frac{3}{298}$ C) $\frac{4}{85}$ D) $\frac{42}{1235}$ E) NOTA

24. Circle O has radius of length 1 and diameter $\overline{AB}$. Point C is outside O, and $\overline{CB}$ intersects O at point D, while $\overline{CA}$ intersects O at point E. If $\angle CAB = 30^\circ$ and $\left|\overline{AC}\right| = 2$, find the value of $\left|\overline{DB}\right|^2$.

A) $4 - 2\sqrt{3}$ B) $\frac{\sqrt{3}}{3}$ C) 1 D) $2 - \sqrt{3}$ E) NOTA

25. The polynomial $f(n)$ gives the *n*th Isaac number, where $f(1) = 2$, $f(2) = 4$, $f(3) = 7$, and $f(6) = 2$. If $f(n)$ is of least possible degree to be consistent with the given information, find the 5th Isaac number.

A) 2 B) 6 C) 8 D) 13 E) NOTA

Tiebreakers

TB1. What number is one-fourth of one-fifth of one-seventh of 70?

TB2. My yellow file folders contain seven sheets of paper each, while my green file folders contain nine sheets of paper each. If I have 18 file folders in all containing a total of 150 sheets of paper, how many of my file folders are yellow?

TB3. If $\cos 36^\circ = \frac{1+\sqrt{5}}{4}$, find the value of $\cos^2 18^\circ$.