

2014 Hoover High School Mathematics Tournament  
Algebra 2 Written Test

1. If  $f(x) = 11x$  and  $x \odot y =$  the greatest common divisor of  $x$  and  $y$ , find the value of  $f(352 \odot 716)$ .  
 A) 44                      B) 88                      C) 132                      D) 176                      E) NOTA
2. At an ultimate Frisbee tournament, there are three expert players for every eight novice players. The players split into teams of seven players, two experts and five novices on each team. By doing this, all of the expert players were placed on a team, but three of the novices did not get to play on a team. How many players total, experts and novices, were at the tournament?  
 A) 44                      B) 66                      C) 88                      D) 110                      E) NOTA
3. Will has \$9. Wilson has the same amount of money as half of Zhey's money plus all of Will's money. Zhey has the same amount of money as Will and Wilson combined. If all three people pool their money, how much money will they have in all?  
 A) \$72                      B) \$84                      C) \$96                      D) \$108                      E) NOTA
4. On any given day, the odds that it will rain are 1:2. Also on any given day, Pranav the Robot has a probability of 75% of getting a computer virus. What are the odds that it is not raining and that Pranav the Robot has a computer virus?  
 A) 3:5                      B) 3:8                      C) 1:8                      D) 1:1                      E) NOTA
5. Solve for  $x$ :  $\log_7(\log_6(\log_5 x)) = 0$   
 A) 210                      B) 3125                      C) 15625                      D) 78125                      E) NOTA
6. The square of one whole number plus the second whole number equals 14. The square of the second whole number plus the first whole number equals 28. What is the sum of the two whole numbers?  
 A) 4                      B) 5                      C) 7                      D) 8                      E) NOTA
7. Simplify:  $\sqrt{-1} \cdot \left(\sqrt{(-1)^3}\right)^3 \cdot \sqrt{(-1)^2} \cdot (\sqrt{-1})^3$   
 A)  $i$                       B)  $-1$                       C)  $-i$                       D) 1                      E) NOTA
8. Round to the nearest integer:  $\frac{\left(\frac{2014}{1008}\right)}{\left(\frac{2012}{1007}\right)}$   
 A) 2                      B) 4                      C) 1                      D) 3                      E) NOTA

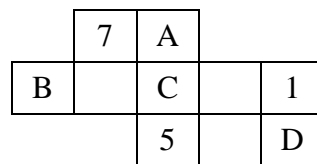
9. Lucy currently has 476 books, and she buys an additional 4 books every day, starting tomorrow. Rachel currently has 986 books, but she gives away 6 books every day, starting tomorrow. In how many days from today will Lucy and Rachel have the same number of books?

- A) 51                      B) 54                      C) 59                      D) 62                      E) NOTA

10. How many asymptotes does the function  $f(x) = \frac{x^3 - 2x^2 - x + 2}{x^2 - 3x + 2}$  have?

- A) 1                      B) 2                      C) 3                      D) 4                      E) NOTA

11. The integers from 1 through 9, inclusive, are to be placed in the diagram to the right, one integer per square; the integers 1, 5, and 7 are already placed. Additionally, no two consecutive integers may be placed in any two squares that touch in any way, including at a corner. In which lettered square must the 3 be placed?



- A) A                      B) B                      C) C                      D) D                      E) NOTA

12. The polynomial  $f(x) = 10x^5 + 13x^4 - 2x^2 - 17x + 6$  is divided by  $g(x) = 2x + 1$  using the Division Algorithm, yielding a quotient polynomial  $q(x)$  and a remainder polynomial  $r(x)$ . Evaluate  $q(2)$ .

- A)  $\frac{175}{2}$                       B)  $\frac{191}{2}$                       C) 175                      D) 191                      E) NOTA

13. Solve for matrix  $X$ :  $\begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix} X + \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 5 & 2 \\ 3 & 4 \end{bmatrix}$

- A)  $\begin{bmatrix} -1 & -1 \\ 4 & 3 \end{bmatrix}$                       B)  $\begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix}$                       C)  $\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$                       D)  $\begin{bmatrix} -11 & -3 \\ 4 & 1 \end{bmatrix}$                       E) NOTA

14. Find the number of consecutive zeros at the end of the base-ten number  $25!$  when it is written as a base-twelve number.

- A) 10                      B) 11                      C) 12                      D) 13                      E) NOTA

15. Joon, Connor, and Daniel are playing a game in which each player secretly selects one of the other two players. All selections are revealed simultaneously, and if all three players are selected, then the three guys win the game; otherwise, they must begin the process all over again. What is the probability that Joon, Connor, and Daniel first win the game on the third try?

- A)  $\frac{1}{4}$                       B)  $\frac{9}{64}$                       C)  $\frac{3}{8}$                       D)  $\frac{75}{512}$                       E) NOTA

16. Find the eccentricity of the conic section with equation  $8x^2 - 3y = 2x + 1$ .

- A)  $\frac{3}{8}$                       B)  $\frac{5}{8}$                       C) 1                      D)  $\frac{8}{3}$                       E) NOTA

17. How many three-digit integers are such that the same digit may not appear all three times, but a digit may appear twice as long as it is in the hundreds' and units' place? A three-digit integer may not begin with a 0.

- A) 512                      B) 625                      C) 729                      D) 900                      E) NOTA

18. In how many ways can six X's be placed in a  $3 \times 3$  grid so that there is at least one X in each row?

- A) 27                      B) 54                      C) 63                      D) 81                      E) NOTA

19. There are eight terms in a sequence, and the sum of any three consecutive terms is 18. If the third term is 8 and the seventh term is 3, determine the value of the fifth term.

- A) 2                      B) 4                      C) 15                      D) 7                      E) NOTA

20. Uzma did some shopping. In the first store she spent  $\frac{1}{4}$  of the money she had, plus \$5. In

the second store she spent  $\frac{1}{2}$  of what she had left, plus \$6. In the last store she spent  $\frac{1}{8}$  of what she had left. If Uzma came home with \$7, how much did she have before she went shopping?

- A) \$33                      B) \$44                      C) \$55                      D) \$66                      E) NOTA

21. Evaluate:  $\sqrt{5 + \sqrt{19 - \sqrt{5 + \sqrt{19 - \dots}}}}$

- A)  $2\sqrt{6}$                       B)  $\sqrt{14}$                       C)  $2\sqrt{3}$                       D) 3                      E) NOTA

22. If the quantity  $x^3 + \frac{1}{x^3}$ , where  $x > 0$ , is twice the quantity  $x + \frac{1}{x}$ , find the greater value of  $x$ .

- A)  $\sqrt{5}$                       B)  $\frac{\sqrt{5}-1}{2}$                       C)  $\frac{\sqrt{5}+1}{2}$                       D)  $\frac{\sqrt{5}+3}{2}$                       E) NOTA

23.  $x = \frac{2}{\sqrt[3]{3}-1}$  is the root of a polynomial  $f(x)$  of least possible degree with integral coefficients whose leading coefficient is 1. Find the coefficient of the  $x$  term in the expansion of  $f(x)$ .

- A) -6                      B) -3                      C) 4                      D) 12                      E) NOTA

24. Find the sum of the series:  $\sum_{n=1}^{\infty} \left( \frac{(2^n + 3^n)^2}{24^n} \right)$

- A)  $\frac{4}{5}$                       B)  $\frac{17}{15}$                       C)  $\frac{22}{15}$                       D)  $\frac{324}{5929}$                       E) NOTA

25. If the equation  $\log_2 x + \log_x 512 = A$ , where  $A > 0$  is real, has only one real solution for  $x$ , find the value of  $A$ .

- A) 4                      B) 6                      C) 8                      D) 9                      E) NOTA

#### Tiebreakers

TB1. Find the number of distinct permutations of the word JUHEEAGRAWAL if the first five letters (JUHEE) must be in some order in the first five positions, and the remaining seven letters (AGRAWAL) must be in some order in the final seven positions.

TB2. What is the length of the latus rectum of the conic section with equation  $\frac{(x+e)^2}{24} + y = \pi$ .

TB3. Find the sum of the cubes of the roots of the equation  $x^3 + 4x - 9 = 0$ .