Geometry

Hoover High School
Mathematics Tournament

March 2, 2013

DIRECTIONS:

1. Do not open this test until you are told to do so.

2. 60 minutes will be allowed for completing this examination. The monitor will keep time. Students must stay in the room for the full 60 minutes.

3. Use a #2 lead pencil.

4. NO calculators, books, notes, or other aides may be used. Scratch paper will be provided; you may not furnish your own. If you need more scratch paper during the test, raise your hand and your monitor will bring it to you. You may write on your test.

5. N. O. T. A. stands for "None of these Answers."

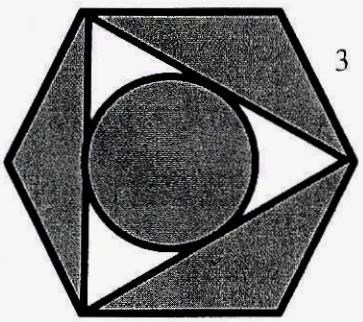
6. You will receive four points for each correct answer minus one point for each incorrect answer on the 25 multiple choice questions. There are three tiebreakers at the end of the test and these will be graded on the basis of 0.1 point for each correct answer. Your score on the written test is the sum of these two scores.

7. Your answers to the tiebreakers should be recorded on your tiebreaker answer sheet.

8. Please give the monitor your answer sheet and your tiebreaker answer sheet before you leave the testing room.

2013 Hoover High School Math Tournament: Geometry Written Test

1. A regular hexagon with side length 3 units is depicted below. The triangle's vertices are alternating vertices of the hexagon. The circle is tangent to the sides of the triangle. Find the area of the shaded region.



A. $\frac{6\pi+27\sqrt{3}}{4}$ B. $\frac{9\pi+27\sqrt{3}}{2}$ C. $\frac{6\pi+27\sqrt{3}}{2}$ D. $\frac{9\pi+27\sqrt{3}}{4}$ E. NOTA

2. What is the total number of diagonals in a 28-gon?

A. 350 B. 364 C. 378 D. 406 E. NOTA

3. Juhee steps out of her Prius and walks 40 yards west in a straight line to Yogurt Mountain. After finishing the best yogurt ever, she attempts to walk 35 yards west on the same straight line to Jason's Deli. Unfortunately, 35 yards was too far to get to Jason's Deli, so she has to turn around and walk 15 yards back. As she is about to enter the deli, a wind from somewhere around the southwest carries her 79 yards then a wind from the opposite direction carries her 18 yards. If Juhee is now directly north of her car, how far is she from it?

A. 10 yards B. 11 yards C. 16 yards D. 25 yards E. NOTA

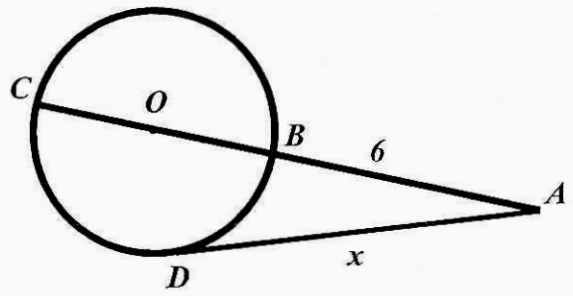
4. If the area of a regular hexagon is $153\sqrt{2}$, what is the area of its inscribed circle?

A. $\frac{\sqrt{102}}{2}\pi$ B. $34\sqrt{6}\pi$ C. $\frac{51\sqrt{6}}{2}\pi$ D. $\sqrt{102}\pi$ E. NOTA

5. The area of a polygon is $196\,in^2$ and its shortest side is 4 in. Find the area of a similar polygon whose shortest side is 6 inches.

A. 130 B. 294 C. 392 D. 441 E. NOTA

6. Line Segment AD is tangent to circle O at Point D. The length of segment AB is labeled as x. What is the area of circle O in terms of x?



A. $\frac{x^2\pi}{144}$ B. $\frac{\left(x^2-36\right)^2\pi}{72}$ C. $\frac{\left(x^2-36\right)^2\pi}{144}$ D. $\frac{x^2\pi}{72}$ E. NOTA

7. In a tunnel with a cross section that is semicircular in shape, a rectangular sign 20 ft. by 2 ft. can be hung horizontally from the top of the tunnel parallel and 18 ft. above the road below. What is the maximum height of the tunnel?

A. $2\sqrt{82}$ B. $20\sqrt{3}$ C. $10\sqrt{5}$ D. $2\sqrt{106}$ E. NOTA

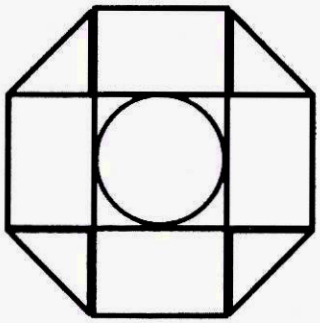
8. Horace the sea barnacle is in the shape of a cylinder with height 11 units and a radius of 4 units. He attempts to start a new under-water fashion trend by growing two congruent, cylindrical "arms" perpendicular from each base. Each arm has a height of 6 units and a radius of 1 unit with hemispheres capping the top. Find the surface area of the new and improved Horace.

A. 176π B. 172π C. 164π D. 150π E. NOTA

9. The sides of a triangle are five, six, and seven. What is the sum of the lengths all three of the altitudes?

A. $\frac{209\sqrt{6}}{35}$ B. $\frac{214\sqrt{6}}{35}$ C. $\frac{418\sqrt{6}}{35}$ D. $\frac{428\sqrt{6}}{35}$ E. NOTA

10. One of the Hoover math team girls orders a chocolate chip waffle at IHOP. When it arrives, it is in the shape of a regular octagon and the chocolate chips form horizontal and vertical lines in the shape similar to that on the Swiss flag. The Hoover math team boys are freaked out by this and throw a pancake on top of it. The pancake forms a perfect circle that is inscribed inside the area of the intersection of the lines as shown below. Assuming that they are two-dimensional, what is the ratio of the area of the waffle to the area of the pancake?

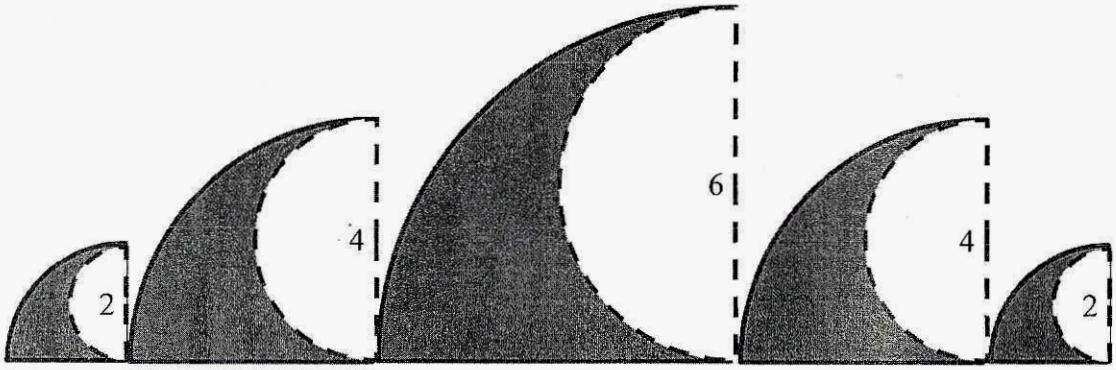


A. $\frac{\left(2+2\sqrt{2}\right)}{\pi}$ B. $\frac{4}{\pi}$ C. $\frac{6}{\pi}$ D. $\frac{\left(8+8\sqrt{2}\right)}{\pi}$ E. NOTA

11. Find the radius of the circle inscribed of a right triangle that has a short leg of 16.

A. 36π B. 49π C. 64π D. 256π E. NOTA

12. The Great Wave of Hoover is the most recognized work of art in the state of Alabama. One of the waves can be represented as a quarter circle with a half circle of half its radius removed. What is the total area of the ocean waves below?



A. $\frac{19\pi}{2}$ B. $\frac{21\pi}{2}$ C. $\frac{35\pi}{2}$ D. $\frac{23\pi}{4}$ E. NOTA

13. What is the sum of the number of Faces, Vertices and Edges of the five Platonic Solids?

A. 50 B. 100 C. 190 D. 210 E. NOTA

14. What is the 2013th triangular number?

A. 2025078 B. 2027091 C. 4050156 D. 4054182 E. NOTA

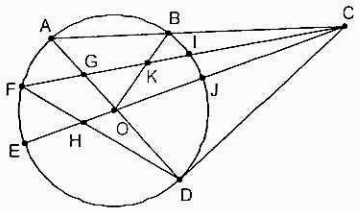
15. Will is taking a 10-question multiple choice math test; however, he forgot to study for it. Each question has four possible choices. What is the probability of randomly choosing the answer for each question and getting them all right?

A. 1/400 B. 1/65536 C. 1/262144 D. 1/1048576 E. NOTA

16. A quadrilateral ABCD is drawn inscribed inside a circle. The consecutive sides of the quadrilateral are of lengths 2, 3, 4 and 5. What is the area of the quadrilateral multiplied by the product of its diagonals?

A. $2\sqrt{30}$ B. $23\sqrt{30}$ C. $43\sqrt{30}$ D. $46\sqrt{30}$ E. NOTA

17. Given: $\odot O$, $m\angle A = 60^\circ$, $m\angle ECF = 10^\circ$, $m\overset{\frown}{AF} = m\overset{\frown}{FE} = 40^\circ$ What is $m\angle FKO$?



A. 90° B. 70° C. 50° D. 30° E. NOTA

18. How many spherical balls, between 100 and 200, can form both a regular triangle in two-dimensions and a regular tetrahedron in three-dimensions?

A. 105 B. 120 C. 153 D. 171 E. NOTA

19. You are traveling directly between the Earth and its moon. Assuming the moon's diameter is 2,000 miles, the Earth's diameter is 8,000 miles and the distance between its centers is 240,000 miles, how far will you be from the moon's center when the moon and the Earth appear to be the same size?

A. 40,000 mi. B. 48,000 mi. C. 60,000 mi. D. 80,000 mi. E. NOTA

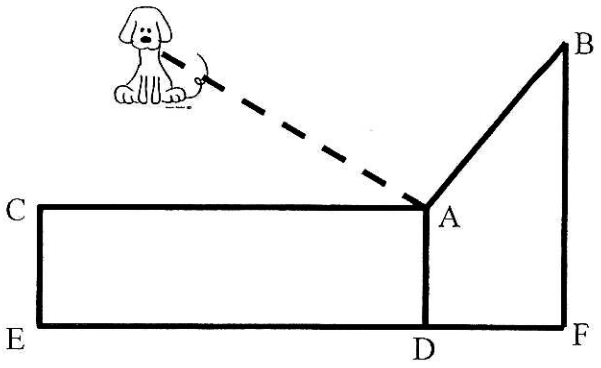
20. You are given two points, A and B, on the larger of two concentric circles. If the line segment AB is tangent to the smaller circle and AB=24, what is the area of the ring formed between the two circles?

A. 25π B. 144π C. 169π D. 225π E. NOTA

21. What is the ratio of the area of a square inscribed in a circle to the area of the square circumscribing the same circle?

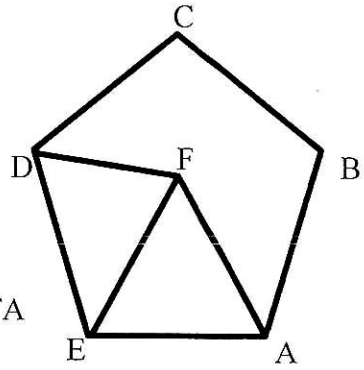
A. 1 : 1 B. 1 : 2 C. 1 : 4 D. 1 : 8 E. NOTA

22. A dog is tied to a rope at point A shown. Its leash is 1.5 times that of length AB. If ADEC is a rectangle with ED=20 and ADFB is a trapezoid with DF=6 and $m\angle CAB=120°$, what is the area the dog can cover? Assume that neither the dog nor the rope can be inside the rectangle or trapezoid.



A. 100π B. 108π C. 123π D. $\frac{400\pi}{3}$ E. NOTA

23. Given regular pentagon ABCDE and equilateral triangle AFE, what is the measure of angle EFD?

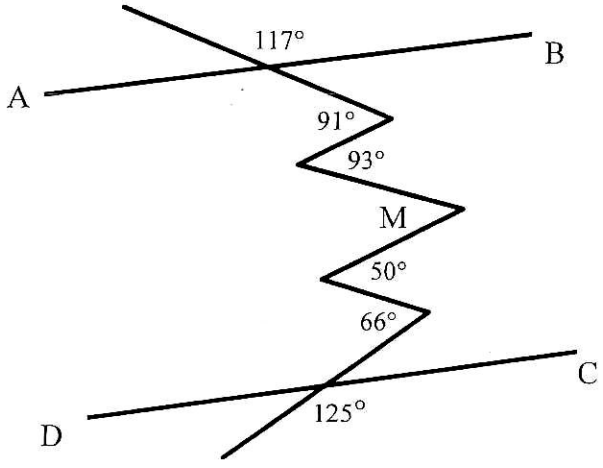


A. 48° B. 50° C. 60° D. 66° E. NOTA

24. If a rhombus is inscribed in a circle where each vertex is on the circle and the circle has a radius of 1, find the area of the rhombus.

A. 1 B. 2 C. 3 D. 4 E. NOTA

25. If line segment AB is parallel to line segment CD, find the measure of angle M.

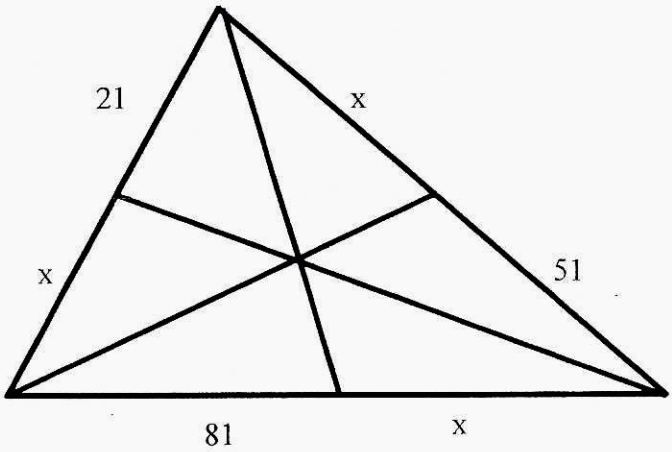


A. 104° B. 100° C. 65° D. 63° E. NOTA

Tiebreakers

TB1: If the hour hand is between 3 and 4 and the angle between the hour and minute hand is 97°, what time is it?

TB2: Solve for the length x below:



TB3: Solve for the angle measure x below