

# Comprehensive

## Hoover High School Mathematics Tournament

March 2, 2013

### DIRECTIONS:

1. Do not open this test until you are told to do so.
2. 60 minutes will be allowed for completing this examination. The monitor will keep time. Students must stay in the room for the full 60 minutes.
3. Use a #2 lead pencil.
4. NO calculators, books, notes, or other aides may be used. Scratch paper will be provided; you may not furnish your own. If you need more scratch paper during the test, raise your hand and your monitor will bring it to you. You may write on your test.
5. N. O. T. A. stands for "None of these Answers."
6. You will receive four points for each correct answer minus one point for each incorrect answer on the 25 multiple choice questions. There are three tiebreakers at the end of the test and these will be graded on the basis of 0.1 point for each correct answer. Your score on the written test is the sum of these two scores.
7. Your answers to the tiebreakers should be recorded on your tiebreaker answer sheet.
8. Please give the monitor your answer sheet and your tiebreaker answer sheet before you leave the testing room.

2013 Hoover High School Mathematics Tournament  
Comprehensive Written Test

1. The first row of Pascal's triangle consists of a single 1. What is the sum of the entries on the 11th row of Pascal's triangle?  
A) 1024      B) 512      C) 256      D) 128      E) NOTA
2. If  $\frac{\sqrt{x^3}}{2} < 12$ , which of the following can be a value of  $x$ ?  
A) -64      B) 9      C) 4      D) 12      E) NOTA
3. Simplify:  $\sin(x+y) \cdot \cos(x-y)$   
A)  $\sin x \cos x + \sin y \cos y$       B)  $\sin x \cos y - \sin y \cos x$       C) 1  
D)  $\sin x + \cos y$       E) NOTA
4. Suppose that  $n$  is a two-digit prime number such that  $2n-1 = x^2$ , where  $x$  is an integer. If  $n$  is as small as possible, find the minimum value of  $n+x$ .  
A) 1      B) 4      C) 8      D) 18      E) NOTA
5. Sunny, Siva, and Madhukar form an Indian shoulder stack by standing on each other's shoulders (and being Indian). Sunny always tells the truth, Siva sometimes tells the truth, and Madhukar never tells the truth. The Indian on top says "The middle Indian is Sunny."; the Indian in the middle says "I'm Siva."; and the Indian on the bottom says "The Indian on the top is Madhukar." Determine which Indian is in the middle.  
A) Sunny      B) Siva      C) Madhukar  
D) unable to be determined based on the given information      E) NOTA
6. Jack has twice as many animal crackers as Janice and Danae combined. Danae has twice as many animal crackers as Janice and 12 fewer animal crackers than Jack. How many animal crackers do Jack, Janice, and Danae have combined?  
A) 18      B) 21      C) 24      D) 27      E) NOTA
7. What is the enclosed area of a triangle with vertices at the points  $(1,0)$ ,  $(2,0)$ , and  $(3,2)$ ?  
A) 1      B) 2      C) 3      D) 10      E) NOTA
8. What is the sum of the eigenvalues of the matrix  $A = \begin{bmatrix} 3 & 8 \\ 2 & 6 \end{bmatrix}$ ?  
A) 4      B) -6      C) 9      D) 17      E) NOTA
9. The product of 7 consecutive integers is 181,440. What is the sum of the least 4 of these 7 integers?  
A) 14      B) 18      C) 22      D) 26      E) NOTA



10. Jacob has a dump truck full of Monster<sub>TM</sub> that he is going to try to feed Mark right before a math tournament. The dump truck is a rectangular prism with dimensions 96 inches by 8 feet by 5 yards. If Jacob forces 12 cubic feet of Monster<sub>TM</sub> down Mark's gullet every 5 minutes, how many hours will the supply of Monster<sub>TM</sub> last?

- A) 400      B) 120      C)  $6\frac{2}{3}$       D) 2      E) NOTA

11. How many points on the graph of  $y = \frac{2(x^2 - 4)}{3(x + 2)^2}$  are not also on the graph of  $y = \frac{2(x - 2)}{3(x + 2)}$ ?

- A) 1      B) 2      C) 3      D) 4      E) NOTA

12. Henry Diggins and Sideshow Mel want to enclose a rectangular field for their pet Will Robersons. If Henry and Mel have 1800 feet of fence, and their field is going to be built against one of the sides of their really, really, ridiculously long building (1,000,000 feet, to be precise), making fence along the building unnecessary, what is the maximum area, in square feet, of Henry and Mel's field? The Will Robersons thank you for maximizing their grazing area.

- A) 360,000      B) 385,000      C) 400,000      D) 405,000      E) NOTA

13. The point  $(1, -1, 0)$  is on a plane, and the vector  $\vec{v} = \langle -8, 9, 12 \rangle$  is perpendicular to the plane. What is the distance from the point  $(2, 3, 2)$  to this plane?

- A)  $\sqrt{21}$       B) 52      C)  $\frac{\sqrt{21}}{17}$       D)  $\frac{52}{17}$       E) NOTA

14. Which of the following can never equal  $\tan \theta$  for some angle  $\theta$ ?

- A)  $\sin \theta$       B)  $\cos \theta$       C)  $\cot \theta$       D)  $\sec \theta$       E) NOTA

15. Dong starts reading a new book on Friday, reading  $\frac{1}{4}$  of its pages. On Saturday, Dong reads  $\frac{1}{4}$  of the remaining pages of the same book. By Sunday, Dong has 162 pages left to read in his new book. How many total pages are in Dong's book?

- A) 272      B) 288      C) 304      D) 320      E) NOTA

16. If  $\sqrt{n + \sqrt{n + \sqrt{n + \dots}}} = i$ , where  $i = \sqrt{-1}$ , find the value of  $|n|$ .

- A)  $\sqrt{2}$       B)  $\sqrt{3}$       C)  $-1 - i$       D)  $1 + i$       E) NOTA

17. Eric's fishing boat leaves a port with a heading of  $60^\circ$  north of east with a speed of 36 knots. At the same time, Xianming's battleship leaves the same port with a heading of  $30^\circ$  south of east with a speed of 15 knots. If one knot is equivalent to one nautical mile per hour, find the distance between the fishing boat and the battleship, in nautical miles, after 6 hours.

- A) 226      B) 230      C) 233      D) 234      E) NOTA

18. David is 84 years old now. Twelve years ago, Andrew was  $\frac{1}{3}$  the age of David. How old is Andrew now?

- A) 24      B) 30      C) 36      D) 42      E) NOTA

19. Nikhil and Khalil want to see who is the il-est (get it? "illest", but "il" because of the endings of their first names? I slay me) math team member. To determine this, Abdullah emcees a rap battle between Nikhil and Khalil. If Nikhil raps at 25 syllables every 3 seconds and Khalil raps 38 syllable every 4 seconds, how many syllables will they lay down together in the final three-minute free-for-all, where both guys rap simultaneously?

- A) 3290      B) 3350      C) 3410      D) 3470      E) NOTA

20. An equilateral triangle is inscribed in a circle with diameter of length 6 cm. Three smaller circles are then drawn with their diameters on each of the sides of the equilateral triangle. Find the exact area, in square cm, of the region enclosed by the three smaller circles that is not also included in the larger circle.

- A)  $\frac{18\sqrt{3}-3\pi}{2}$       B)  $\frac{9\pi-54\sqrt{3}}{8}$       C)  $\frac{45\pi+27\sqrt{3}}{4}$       D)  $\frac{9\pi+54\sqrt{3}}{8}$       E) NOTA

21. Joey's pet boxing kangaroo just had two babies, aptly named "Joey's joeys". After 2 months, those two joeys had two each of their own joeys, aptly titled "Joey's joeys' joeys". After 2 more months, those four joeys had two each of their own joeys, aptly titled "Joey's joeys' joeys' joeys". If each kangaroo or joey can only have two joeys itself, and if this naming scheme were to continue, with each joey or kangaroo having two joeys of its own after 2 months, and none of the kangaroos or joeys died, how many total kangaroos (adults and joeys) does Joey now have through the generation of "Joey's joeys' joeys' joeys' joeys' joeys' joeys' joeys"?

- A) 127      B) 255      C) 511      D) 1023      E) NOTA

22. What is the sum of the last 3 digits in the expansion of  $7^{2013}$ ?

- A) 7      B) 10      C) 11      D) 21      E) NOTA

23. The nine students in Comp Team class are sitting at a round table. If Slush Mutha, who is one of the nine students, has her own special seat whose position at the table is fixed, in how many distinct ways can the students in Comp Team class be seated at the round table?

- A) 40,320      B) 3,628,800      C) 362,880      D) 5040      E) NOTA

24.  $a$ ,  $b$ , and  $c$  are complex numbers such that  $a^2 + b^2 + c^2 = 15$ ,  $a^3 + b^3 + c^3 = 31$ , and  $abc = 15$ . Find the possible positive value of  $a + b + c$ .

- A) 7      B) 8      C) 9      D) 10      E) NOTA

25. Find the vertex of the parabola given by the equation  $x^2 + 4xy + 4y^2 - 30x - 90y + 450$ .

- A)  $\left(\frac{21\sqrt{5}}{5}, \frac{3\sqrt{5}}{10}\right)$       B)  $\left(\frac{18}{5}, \frac{87}{10}\right)$       C)  $\left(\frac{20\sqrt{5}}{5}, \frac{3\sqrt{5}}{5}\right)$       D)  $\left(\frac{25\sqrt{2}}{2}, \frac{15\sqrt{2}}{2}\right)$       E) NOTA

## Tiebreakers

TB1. What 7-digit palindrome satisfies the following conditions?

- (A) No digit appears more than twice.
- (B) Zero is not used in the palindrome.
- (C) The last digit is three times the second digit.
- (D) The third digit minus the sixth digit equals the middle digit.
- (E) The sum of all seven digits is 41.

TB2. Some of Machiavelli's friends were going to contribute 4 lira each to buy him a birthday cake. However, three friends backed out, making them enemies, and you know what Machiavelli does to his enemies, don't you? The true (remaining) friends had to chip in an extra 2 lira each to make up the difference. How many people were originally going to contribute to the cost of the birthday cake for Machiavelli?

TB3. What is the fewest number of people that must be chosen at random from a group of 1000 people to ensure that at least four people chosen are born in the same month?