Algebra II
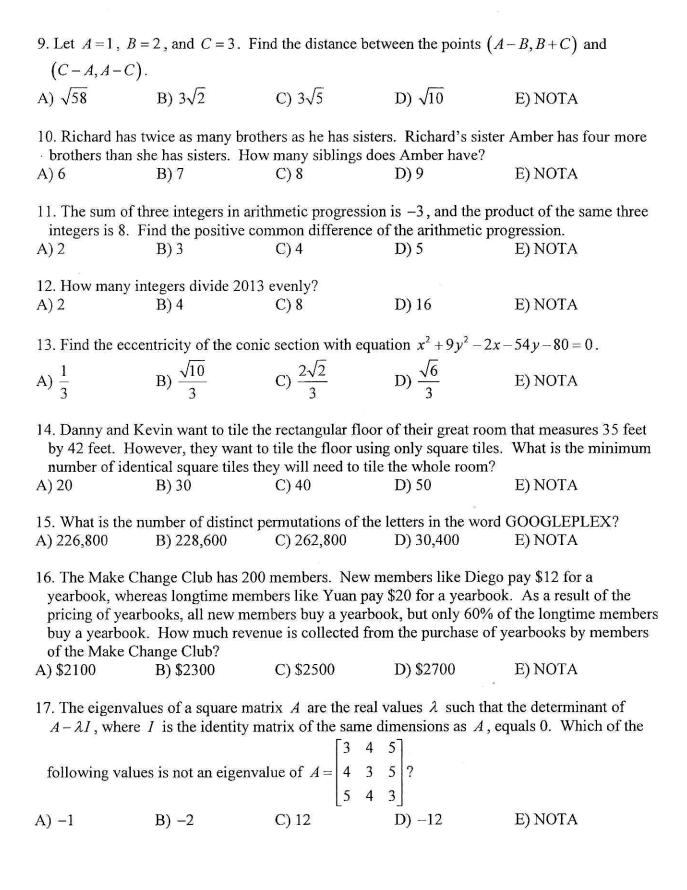
Hoover High School
Mathematics Tournament

March 2, 2013

DIRECTIONS:

1. Do not open this test until you are told to do so.

2. 60 minutes will be allowed for completing this examination. The monitor will keep time. Students must stay in the room for the full 60 minutes.

3. Use a #2 lead pencil.

4. NO calculators, books, notes, or other aides may be used. Scratch paper will be provided; you may not furnish your own. If you need more scratch paper during the test, raise your hand and your monitor will bring it to you. You may write on your test.

5. N. O. T. A. stands for "None of these Answers."

6. You will receive four points for each correct answer minus one point for each incorrect answer on the 25 multiple choice questions. There are three tiebreakers at the end of the test and these will be graded on the basis of 0.1 point for each correct answer. Your score on the written test is the sum of these two scores.

7. Your answers to the tiebreakers should be recorded on your tiebreaker answer sheet.

8. Please give the monitor your answer sheet and your tiebreaker answer sheet before you leave the testing room.

1. Simplify: $\dfrac{\dfrac{2}{9}+\dfrac{4}{15}}{\dfrac{4}{5}+\dfrac{5}{6}}$

A) $\dfrac{2}{7}$ B) $\dfrac{43}{147}$ C) $\dfrac{44}{147}$ D) $\dfrac{15}{49}$ E) NOTA

2. If $y=mx+b$, where m and b are real numbers, is the equation of the line through the points $(7,-3)$ and $(3,5)$, find the product $m\cdot b$.

A) 22   B) –22   C) 2   D) –2   E) NOTA

3. Find the sum of the coefficients in the expansion and simplification of $(x+2)^{6}-(x+2)^{5}$.

A) 32   B) 64   C) 243   D) 486   E) NOTA

4) Ishant has a certain number of fish and fishbowls in his pet store. If Ishant puts one fish in each bowl, there are four fish left over. If, however, he puts three fish in each bowl (and places all of the fish into bowls), there are six bowls left over. How many fishbowls does Ishant have in his pet store?

A) 9   B) 11   C) 13   D) 15   E) NOTA

5. How many integers satisfy the inequality $\left|\dfrac{2}{3}x-4\right|\le\dfrac{58}{9}$?

A) 17   B) 18   C) 19   D) 20   E) NOTA

6. When the denominator is rationalized, the quantity $\dfrac{30}{\sqrt{3}+\sqrt{7}+\sqrt{10}}$ can be written as

$\dfrac{A\sqrt{3}+B\sqrt{7}+C\sqrt{210}}{7}$, where A, B, and C are integers. Find the value of $A+B+C$.

A) 45   B) 48   C) 51   D) 54   E) NOTA

7. Marshall distributes 100 individually wrapped pieces of candy into five dishes, marked A, B, C, D, and E. Thirty-eight total candies are in dishes A and B, 26 total candies are in dishes B and C, 47 total candies are in dishes C and D, and 53 total candies are in dishes D and E. How many total candies are in dishes B and D?

A) 42   B) 48   C) 55   D) 61   E) NOTA

8. What is the probability that Alex rolls a sum of 5 or less when he rolls three fair, standard six-sided dice?

A) $\dfrac{5}{12}$ B) $\dfrac{5}{18}$ C) $\dfrac{5}{54}$ D) $\dfrac{5}{108}$ E) NOTA

9. Let $A=1$, $B=2$, and $C=3$. Find the distance between the points $(A-B, B+C)$ and $(C-A, A-C)$.

A) $\sqrt{58}$ B) $3\sqrt{2}$ C) $3\sqrt{5}$ D) $\sqrt{10}$ E) NOTA

10. Richard has twice as many brothers as he has sisters. Richard's sister Amber has four more brothers than she has sisters. How many siblings does Amber have?

A) 6   B) 7   C) 8   D) 9   E) NOTA

11. The sum of three integers in arithmetic progression is -3, and the product of the same three integers is 8. Find the positive common difference of the arithmetic progression.

A) 2   B) 3   C) 4   D) 5   E) NOTA

12. How many integers divide 2013 evenly?

A) 2   B) 4   C) 8   D) 16   E) NOTA

13. Find the eccentricity of the conic section with equation $x^2 + 9y^2 - 2x - 54y - 80 = 0$.

A) $\dfrac{1}{3}$ B) $\dfrac{\sqrt{10}}{3}$ C) $\dfrac{2\sqrt{2}}{3}$ D) $\dfrac{\sqrt{6}}{3}$ E) NOTA

14. Danny and Kevin want to tile the rectangular floor of their great room that measures 35 feet by 42 feet. However, they want to tile the floor using only square tiles. What is the minimum number of identical square tiles they will need to tile the whole room?

A) 20   B) 30   C) 40   D) 50   E) NOTA

15. What is the number of distinct permutations of the letters in the word GOOGLEPLEX?

A) 226,800   B) 228,600   C) 262,800   D) 30,400   E) NOTA

16. The Make Change Club has 200 members. New members like Diego pay \$12 for a yearbook, whereas longtime members like Yuan pay \$20 for a yearbook. As a result of the pricing of yearbooks, all new members buy a yearbook, but only 60% of the longtime members buy a yearbook. How much revenue is collected from the purchase of yearbooks by members of the Make Change Club?

A) \$2100   B) \$2300   C) \$2500   D) \$2700   E) NOTA

17. The eigenvalues of a square matrix A are the real values λ such that the determinant of $A - \lambda I$, where I is the identity matrix of the same dimensions as A, equals 0. Which of the following values is not an eigenvalue of $A = \begin{bmatrix} 3 & 4 & 5 \\ 4 & 3 & 5 \\ 5 & 4 & 3 \end{bmatrix}$?

A) -1 B) -2 C) 12   D) -12 E) NOTA

18. Jeremy's favorite toy is a solid wooden cube. To jazz it up, Jeremy paints his favorite toy solid black, then cuts it into 216 identical smaller cubes. How many faces of the smaller identical cubes are not painted black?

A) 125   B) 216   C) 750   D) 1080   E) NOTA

19. Yilan's safe has a seven-digit combination that uses each of the digits 1 through 7 with no repeated digits. Yilan can't remember the combination, but she does remember that 534 is somewhere in the combination, consecutively, in that order. What is the maximum number of arrangements Yilan will have to try to rediscover the combination to her safe?

A) 30   B) 60   C) 120   D) 5040   E) NOTA

20. Find the coefficient of x^3 in the expansion of $\left(x^2 - x + 2\right)^6$.

A) −400   B) 400   C) −80   D) 80   E) NOTA

21. In how many four-digit numbers is the sum of the digits equal to 32?

A) 32   B) 35   C) 38   D) 41   E) NOTA

22. Find the range of the function $f(x) = \dfrac{x^2 - 2}{x^2 - 4}$.

A) $(-\infty,-1)\cup(-1,\infty)$ B) $(-\infty,1)\cup(1,\infty)$ C) $(-\infty,0)\cup(1,\infty)$

D) $(-\infty,0.5)\cup(1,\infty)$ E) NOTA

23. Find the sum of all radian values of x in the interval $[-2\pi, 2\pi]$ such that

$$\frac{\cos(5x) - \cos(3x)}{\sin(5x) + \sin(3x)} = \sqrt{3}.$$

A) $\dfrac{\pi}{3}$ B) $\dfrac{2\pi}{3}$ C) $\dfrac{4\pi}{3}$ D) $\dfrac{7\pi}{3}$ E) NOTA

24. Abhay's bug collection contains only ants and spiders. One of the ants, Gimpy, only has 5 legs while all the other ants have 6 legs. Two of the spiders, Charlotte and Manspider, also have strange leg amounts: Charlotte has 9 legs while Manspider only has 2 legs. All of the other spiders have 8 legs. If Abhay's bug collection contains 37 total bugs with 248 total legs, how many 8-legged spiders does Abhay have in his collection?

A) 14   B) 15   C) 16   D) 17   E) NOTA

25. Cooking Mama can cook 5 meals in 3 hours, Cooking Papa can cook 5 meals in 4 hours, and Cooking Baby can cook 5 meals in 8 hours. If the three members of the Cooking family are given 24 hours to prepare as many meals as is possible, at which point Isaac begins eating meals at the rate of 5 meals every 2 hours, how many hours will it take Isaac to eat until there are no meals remaining? Assume that all three members of the Cooking family cook independently and simultaneously.

A) 24   B) 27   C) 32   D) 34   E) NOTA

Tiebreakers

TB1. There are 3×10^{27} molecules in a basin whose volume is 7.5×10^{6} cubic centimeters. What is the average number of molecules per cubic centimeter in this basin? Write your answer in scientific notation.

TB2. Find the value of x that satisfies the equation $\dfrac{55\cdot14\cdot x}{77\cdot13}=30$.

TB3. Find the real root of the polynomial $x^{3}+9x^{2}+27x+25=0$.