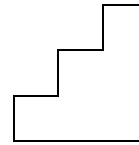


2012 Hoover High School Mathematics Tournament  
Comprehensive Written Test

1. In the drawing of a set of stairs to the right, all line segments meet at right angles, and each vertical and horizontal segment forming the steps is 2 units long. Find the ratio of the perimeter of the drawing to the enclosed area of the drawing.



- A)  $\frac{1}{2}$       B) 1      C)  $\frac{3}{2}$       D) 2      E) NOTA

2. If it takes Abdullah 5 days to write the lyrics to Justin Bieber's hit song "Omaha Mall" by himself, and it takes Jacob 3 days to write the lyrics to "Omaha Mall" by himself, how many hours would it take both of them, working independently but together, to write down the lyrics to "Omaha Mall"?

- A) 48      B) 96      C) 45      D) 90      E) NOTA

3. Among all distinct permutations of the letters in the name SUSHMITHA, how many include the name SUSMITA, written from left to right with letters in consecutive positions?

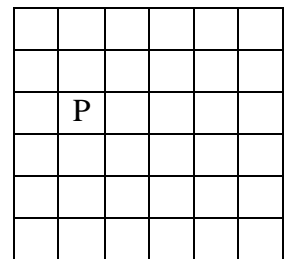
- A) 3      B) 6      C) 12      D) 24      E) NOTA

4. The world is ending in 11 hours, and Nicolas Cage is the only person who is aware of this tragedy. He can only tell one person at a time, and it takes 1.5 hours for the person he is telling to fully comprehend the gravity of the situation. Any person who knows of this situation can in turn tell only one person at a time, again taking a full 1.5 hours for that new person to fully comprehend the gravity of the situation. How many people will fully comprehend the gravity of the world ending by the time that it does in fact end?

- A) 32      B) 128      C) 255      D) 256      E) NOTA

5. In the  $6 \times 6$  grid to the right, a pawn is placed in the square marked with a P. How many squares whose sides are on the lines of the grid do not contain the pawn?

- A) 52      B) 57      C) 64      D) 69      E) NOTA



6. Simplify:  $6 - 4 \div 2 - (-1 + 3)^2$

- A) 6      B) -3      C) 0      D) 5      E) NOTA

7. Rebecca Black wants to know the frequency of the pitch of her normal, beautiful voice. To mess with her, Joshua tells her that on Friday, Friday, Friday, he will let her know, but he plans to tell her the frequency in binary. If he tells her that her pitch is 111110100, what is Rebecca Black's pitch frequency in base 10?

- A) 64      B) 6      C) 22222020      D) 500      E) NOTA

8. At what time between 11 pm and midnight does the smaller angle between the hour and minute hands on a standard clock have a measure of  $22^\circ$ ?

- A) 11:55 pm      B) 11:56 pm      C) 11:57 pm      D) 11:58 pm      E) NOTA

9. Eric's favorite five-digit number is the number  $ABCDE$ . If you multiplied Eric's favorite five-digit number by 4, you'd get Jack's favorite five-digit number,  $EDCBA$ . Find Olga's favorite five-digit number,  $CDBEA$ .

- A) 97182      B) 54632      C) 32182      D) 64382      E) NOTA

10. Siva spends half of his money on a cricket bat, then increases his cash flow by 300% by releasing a cover version of "Why This Kolaveri Di". If Siva now has \$1200, how much money did he have before he bought the cricket bat?

- A) \$400      B) \$500      C) \$800      D) \$1000      E) NOTA

11. What is the sum of the five least nonnegative even integers and the four greatest odd negative integers?

- A) 4      B) 8      C) 10      D) 14      E) NOTA

12. How many lattice points satisfy the equation  $|x| + |y| = 5$ ?

- A) 12      B) 20      C) 24      D) 36      E) NOTA

13. Find the value of  $\lceil F(F(5)) \rceil$ , where  $F(x) = \frac{9x^2 - 25x + 50}{25}$  and  $\lceil x \rceil$  represents the least integer greater than or equal to  $x$ .

- A) 6      B) 7      C) 8      D) 9      E) NOTA

14. What is the sine of the smaller of the two angles created by the vectors  $\langle 4, 1, 8 \rangle$  and  $\langle 8, 24, 6 \rangle$ ?

- A)  $\frac{\sqrt{65}}{9}$       B)  $\frac{4}{9}$       C)  $\frac{\sqrt{5}}{3}$       D)  $\frac{2}{3}$       E) NOTA

15. Dieter Laser would like to line up you and five friends (Todd, Dong, Madhukar, Nicole, and Sunny) in a sequence for an experiment he is conducting. You refuse to be next to Madhukar, and Todd cannot be at the front of the line. How many different sequences of people are possible?

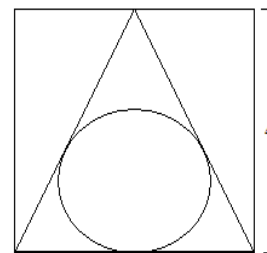
- A) 288      B) 312      C) 408      D) 600      E) NOTA

16. Find the sum of the values of  $x$  such that  $9(2)^{2x+3} - (2)^{4x} = 512$ .

- A) 3      B) 72      C)  $\frac{9}{2}$       D) 9      E) NOTA

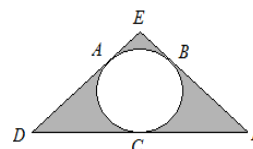
17. Joey's race car enthusiast club logo looks like the picture to the right, with a circle inscribed in an equilateral triangle inscribed in a rectangle with height 4, such that one side of the triangle coincides with the base of the rectangle. Find the area enclosed by the circle.

- A)  $\frac{25\pi}{16}$       B)  $\frac{16\pi}{9}$       C)  $\frac{9\pi}{4}$       D)  $4\pi$       E) NOTA



18. Henry's race car enthusiast club logo looks like the picture to the right, with a circle inscribed in a triangle with tangent points  $A$ ,  $B$ , and  $C$ . If  $m\angle ABC = 225^\circ$ ,  $|\overline{CD}| = |\overline{CF}|$ , and the radius of the circle has length 10, what is the area of the shaded region?

- A)  $600 - 300\sqrt{2} - 100\pi$       B)  $225 + 100\sqrt{2} - 100\pi$       C)  $400 + 300\sqrt{2} - 100\pi$       D)  $300 + 200\sqrt{2} - 100\pi$   
E) NOTA



19. A parabola is given by the equation  $x^2 + 4xy + 4y^2 - 30x - 90y + 450 = 0$ . Among all points on this parabola, the maximum  $x$ -value of a point is  $a$  while the minimum  $y$ -value of a point is  $b$ . Find the value of  $a + b$ .

- A)  $\frac{45}{4}$       B)  $\frac{55}{4}$       C)  $\frac{65}{4}$       D)  $\frac{75}{4}$       E) NOTA

20. Mark and Khalil attend part of a Bear Scouts meeting that is two hours in length, from 7 pm until 9 pm. Mark and Khalil arrive at random times, but Mark won't arrive after 8:45 pm while Khalil won't arrive before 7:30 pm. If both Bear Scouts stay from the time they arrive at the meeting until the end of the meeting, what is the probability that the sum of the amounts of time that each of the two is at the meeting is at least two hours total?

- A)  $\frac{2}{5}$       B)  $\frac{3}{7}$       C)  $\frac{4}{9}$       D)  $\frac{5}{11}$       E) NOTA

21. Wesley draws three small congruent circles inside a larger circle such that each of the three congruent circles are externally tangent to each other and are all three tangent to the larger circle. Find the ratio of the area enclosed by the large circle to the area enclosed by one of the three smaller congruent circles.

- A)  $\frac{4}{15}$       B)  $\frac{3}{16}$       C)  $\frac{7+4\sqrt{3}}{3}$       D)  $\frac{13+4\sqrt{3}}{6}$       E) NOTA

22. Consider positive integer value  $n$ . It is possible to find three distinct positive factors  $x$ ,  $y$ , and  $z$  of  $n-1$  such that  $x+y+z=n$ . 13 is one such value since 6, 4, and 3 are factors of 12 whose sum is 13. Which of the following integers could also be a value of  $n$  with this property?

- A) 25      B) 29      C) 37      D) 41      E) NOTA

23. Let  $F_n$  be the  $n$ th Fibonacci number ( $F_1 = F_2 = 1$ ,  $F_n = F_{n-1} + F_{n-2}$  for  $n \geq 3$ ). Find the positive value of  $x$  satisfying  $\sum_{i=1}^{\infty} x^i F_i = 1$ .

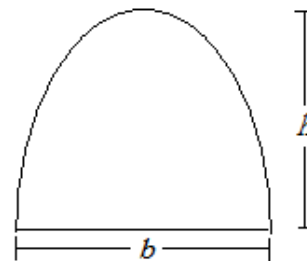
- A)  $\frac{\sqrt{5}+1}{4}$       B)  $\sqrt{3}-1$       C)  $\frac{\sqrt{5}-1}{4}$       D)  $\sqrt{2}-1$       E) NOTA

24. From the interior of equilateral  $\triangle ABC$ , point  $P$  is chosen such that  $|AP| = 5$ ,  $|BP| = 4$ , and  $|CP| = 3$ . Find the area enclosed by equilateral  $\triangle ABC$ .

- A)  $\frac{37\sqrt{3}}{4}$       B)  $\frac{45+34\sqrt{3}}{4}$       C)  $\frac{36+25\sqrt{3}}{4}$       D)  $16\sqrt{3}$       E) NOTA

25. Given a parabolic sector like the one shown, if  $b$  is the length across the bottom of the sector and  $h$  is the height of the sector, the enclosed area is given by the formula  $A = \frac{2}{3}bh$ . Armed with this information, given the quadratic equation  $5x^2 + bx + c = 0$ , where  $b$  and  $c$  are real numbers in the interval  $[0, 5]$ , find the probability that the quadratic equation has one or two real solutions.

- A)  $\frac{1}{12}$       B)  $\frac{1}{5}$       C)  $\frac{5}{8}$       D)  $\frac{3}{5}$       E) NOTA



#### Tiebreakers

TB1. Find the sum of the reciprocals of the positive integral factors of 8128.

TB2. Dr. Kustos decided that he was going to listen to the Flaming Lips' song "7 Skies H3", a song that is exactly 24 hours in length, in one 24-hour period. However, he forgot to account for bathroom breaks (3 of them, 10 minutes each), meals (3 of them, 30 minutes each), and stretch breaks to keep his blood circulating (6 of them, 15 minutes each). If he began listening to the song at midnight on Halloween, when it was released, and he paused the song for each of the breaks, at what time on November 1 did he finish?

TB3. The Man from Another Place has eight coins in his pocket that total \$1.85. How many distinct possible sets of coins could the Man from Another Place possibly have? Assume U.S. currency of pennies, nickels, dimes, quarters, and half-dollars.