

2012 Hoover High School Mathematics Tournament
Algebra 2 Written Test

1. Solve for x : $\frac{27 \cdot 28 \cdot x}{42 \cdot 45} = 26$
 A) 65 B) 60 C) 55 D) 50 E) NOTA
2. Nikhil's gym container holds 50 balls of two sizes and two colors. Twenty-two balls are large and red, 12 balls are small, and 26 balls are green. How many of the balls are both small and green?
 A) 8 B) 9 C) 10 D) 11 E) NOTA
3. If Cavan can fill 62.5% of a bucket with water in one minute, working at the same rate, how many seconds will it take to fill the rest of the bucket?
 A) 24 B) 36 C) 84 D) 96 E) NOTA
4. When Layla woke up last night, she noticed that the hour hand of her wall clock was pointing directly at the 23-minute mark. What time did Layla wake up?
 A) 4:24 B) 4:30 C) 4:36 D) 4:44 E) NOTA
5. How many distinct permutations of the letters in the word REAPPEAR have the feature that all the identical letters are adjacent to each other in the permutation?
 A) 24 B) 120 C) 1260 D) 2520 E) NOTA
6. Will buys packages of Kit Kat candy bars at \$7 for a box of 20 which he later turns around and sells. He receives as much for 4 Kit Kats as he pays for 5 Kit Kats. What is his selling price for a box of 20 Kit Kats?
 A) \$7.50 B) \$8.00 C) \$8.25 D) \$8.75 E) NOTA
7. Simplify: $\log_{18} 4 + \log_{18} 81$
 A) $\log_{18} 85$ B) $\frac{1}{2}$ C) $\log_{85} 18$ D) 2 E) NOTA
8. How many integers satisfy the inequality $65 < x^2 < 120$?
 A) 2 B) 3 C) 4 D) 5 E) NOTA
9. Given that $f(x^2 + 1) = 2x + f(x)$ for all real x and $f(0) = 7$, find the value of $f(5)$.
 A) 9 B) 13 C) 11 D) 23 E) NOTA
10. If $8x - 4y = 7$ and $x - 2y = 5$, find the value of $x + y$.
 A) $-\frac{15}{7}$ B) $-\frac{13}{4}$ C) $\frac{15}{7}$ D) $\frac{13}{4}$ E) NOTA
11. Find the sum of all positive integer values of n such that $10 < n! < 10!$.
 A) 0 B) 4 C) 23 D) 39 E) NOTA
12. In a van, there are two seats in the front (driver and shotgun), two seats in the middle, and one seat in the back. Five people are to be seated in the van, but only three people can drive, and one person cannot sit in either seat in the front. How many total ways are there for the five people to sit in the van?
 A) 120 B) 54 C) 72 D) 36 E) NOTA

13. Find the sum of the entries of matrix X if $\begin{bmatrix} -2 & -7 \\ 1 & 3 \end{bmatrix} X = \begin{bmatrix} 0 & 1 & -6 \\ -3 & -2 & 1 \end{bmatrix}$.
- A) 30 B) 24 C) -30 D) 24 E) NOTA
14. Find the value of $(\csc 15^\circ + \csc 75^\circ)^2$.
- A) 24 B) 12 C) 48 D) 36 E) NOTA
15. Find the units digit of the quantity $\sum_{n=1}^9 \sum_{m=1}^9 m^n$.
- A) 7 B) 3 C) 1 D) 0 E) NOTA
16. Consider the ellipse with equation $\frac{(x+5)^2}{9} + \frac{(y-2)^2}{6\sqrt{2}} = 1$. A triangle is formed so that one of its vertices is on a directrix of the ellipse while the other vertices are the endpoints of the latus rectum closer to that directrix. Find the area enclosed by the triangle.
- A) $6\sqrt{6} + 6\sqrt{3}$ B) $7\sqrt{6} + 7\sqrt{3}$ C) $8\sqrt{6} + 8\sqrt{3}$ D) $9\sqrt{6} + 9\sqrt{3}$ E) NOTA
17. Xianming can paint a house in 20 hours. Xianming works for 1 hour painting the house, at which point David shows up to help. David can paint the same house in 30 hours. After working together and independently on painting the house for 2 more hours, Janice shows up to help. Janice can paint the same house in 40 hours. The three of them continue to paint together and independently until the house is completely painted. How many hours total did it take for the house to get painted?
- A) 9 B) $\frac{120}{13}$ C) 10 D) $\frac{133}{13}$ E) NOTA
18. The polynomial $f(x) = 3x^4 - 8x^3 - 5x^2 + 16x - 5$ can be factored into two quadratics $ax^2 + bx + c$ and $dx^2 + ex + f$, both with integer coefficients, and both quadratics are irreducible over integers. Find the value of $a+b+c+d+e+f$.
- A) -2 B) -1 C) 1 D) 2 E) NOTA
19. If $x = \frac{\sqrt{6} + \sqrt{2}}{\sqrt{6} - \sqrt{2}}$, find the value of $\frac{x^2 + 1}{x}$.
- A) 2 B) 4 C) $2\sqrt{3}$ D) $4\sqrt{3}$ E) NOTA
20. Andrew drove from Hoover High School to Beef ‘O’ Brady’s at a constant rate of 30 miles per hour. He made his return trip along the same route, but at a faster constant rate that was also an integer. If Andrew’s average round trip rate was an integer number of miles per hour, what is the minimum speed that Andrew could have had during his return trip?
- A) 35 B) 38 C) 42 D) 45 E) NOTA
21. Compute 19 times the value of $12 + \frac{1}{2 + \frac{1}{1 + \frac{1}{2 + \frac{1}{1 + \frac{1}{1}}}}}$.
- A) 233 B) 235 C) 237 D) 239 E) NOTA

22. Two trains, each of length 1 mile, are traveling in opposite directions along two sets of straight tracks, both toward a tunnel that is 2 miles long. The front of the first train is 3 miles from the end of the tunnel closest to it, and that train is traveling 30 miles per hour. The front of the second train is 8 miles from the end of the tunnel closest to it. In order for any part of the two trains to be in the tunnel at the same time, this second train must be traveling faster than x miles per hour and slower than y miles per hour. Find the value of $x + y$.

- A) 135 B) 140 C) 145 D) 150 E) NOTA

23. Two bags of M&M's contain only brown and blue M&M's. In their original state, the first bag contains 8 blue and 2 brown M&M's while the second bag contains 5 blue and 7 brown M&M's. An M&M is taken from the first bag and placed into the second bag without looking at it; then, an M&M is taken from the second bag and placed into the first bag without looking at it. What is the probability that the two bags contain the exact same color distributions as in their original state?

- A) $\frac{32}{65}$ B) $\frac{33}{65}$ C) $\frac{34}{65}$ D) $\frac{7}{13}$ E) NOTA

24. Find the value of $\sqrt{2012 \cdot 2013 \cdot 2014 \cdot 2015 + 1}$.

- A) 4,054,169 B) 4,054,171 C) 4,054,179 D) 4,054,181 E) NOTA

25. The Fastest Rower Ever is rowing his canoe upstream in a straight river. He rows for 40 miles when his hat falls off into the river. He continues to row upstream for 10 more minutes before he realizes his hat fell off. He immediately turns around to retrieve his hat, at which point his other hat falls off into the river. He retrieves the first hat at his initial starting point, then turns around immediately and retrieves the second hat 30 miles upstream from his original starting point. How fast does The Fastest Rower Ever row in still water, in miles per minute?

- A) 5 B) 6 C) 7 D) 8 E) NOTA

Tiebreakers

TB1. Given $a_n = \begin{cases} \frac{5}{2^n}, & \text{if } n \text{ is even} \\ \frac{2}{3^n}, & \text{if } n \text{ is odd} \end{cases}$, find the value of $\sum_{n=1}^{\infty} a_n$.

TB2. Find the determinant of the matrix $\begin{vmatrix} 1009 & 1010 \\ 2012 & 2013 \end{vmatrix}$.

TB3. Find the greatest root of the polynomial $f(x) = x^4 - 4x^3 + 4x^2 - 1$.