2011 Vestavia Hills High School
Mathematics Tournament
Algebra I Written Examination

1. Simplify $\dfrac{x^4-81}{x^3-27} \times \dfrac{x^2-10x+21}{x^4-6x^2-27} \div \dfrac{x-7}{x^2+3}$.

   A. $\dfrac{x+3}{x^2+9}$ B. $\dfrac{(x-7)^2(x^2+9)}{(x-7)^2(x^2+3x+9)}$ C. $\dfrac{1}{3x}$ D. $\dfrac{x^2+9}{x^2+3x+9}$ E. NOTA

2. There are 10 boys and 10 girls in a club. If there are four officer positions—one president, one vice president, one secretary, and one coordinator—how many combinations of officers are there if the president must be a boy and the secretary must be a girl? *President and secretary are chosen first*

   A. 3060 B. 18000 C. 30600 D. 45000 E. NOTA

3. What is the equation of the line that passes through (1, 4) and is perpendicular to a line whose equation is $y=\dfrac{2}{3}x+5$?

   A. $y=\dfrac{2}{3}x+\dfrac{10}{3}$ B. $y=\dfrac{2}{3}x+1$ C. $y=-\dfrac{3}{2}x+7$ D. $y=-\dfrac{3}{2}x+\dfrac{11}{2}$ E. NOTA

4. Simplify $\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{4^{4!}}}}}}$, where ! denotes the factorial.

   A. $4^{\frac{3}{4}}$ B. $4^{\frac{2}{3}}$ C. $4^{\frac{1}{8!}}$ D. $4^{\frac{3}{2}}$ E. NOTA

5. A man has fallen into a 100 ft ditch. While trying to climb out, he falls 1.5 ft for every 3 ft he climbs. After how many attempts will he be able to get out?

   A. 65 B. 66 C. 77 D. 68 E. NOTA

6. The average of two numbers is 15. The average of five other numbers is –7. What is the average of all seven numbers?

   A. 4 B. $-\dfrac{5}{7}$ C. $\dfrac{5}{7}$ D. $\dfrac{4}{7}$ E. NOTA

7. If 160 concert tickets were sold for $1120, how many unsupervised children will be at the concert if adult tickets cost $10 and children's tickets cost $6? Assume each adult will supervise two children.

   A. 40 B. 80 C. 100 D. 120 E. NOTA

8. If each of *AB* and *CD* separately represent any two-digit number from 00-98, what is any one ordered pair (*AB*, *CD*) that would make both of the following statements true?
   - Farhan was born in 19*AB* and turns *CD* years old in 1998.
   - Evan was born in 19*CD* and turns *AB* years old in 1998.

   A. (47, 49) B. (48, 49) C. (90, 08) D. (46, 50) E. NOTA

9. What is the sum of the mean and median of the elements in the fifth row of Pascal's triangle if the first row contains one element?

   A. 10 B. $\dfrac{31}{3}$ C. $\dfrac{107}{7}$ D. $\dfrac{36}{5}$ E. NOTA

10. Yash, Yue, Gene, Wendy, Farhan, and Evan are sitting at a circular table. How many distinct ways can they be arranged?

    A. 30   B. 60   C. 120   D. 240   E. NOTA

11. If $3^{xyz} = 9$, and $4^{xy} = 8$, what is z ?

    A. $\dfrac{3}{2}$ B. $\dfrac{3}{4}$ C. $\dfrac{4}{3}$ D. $\dfrac{2}{3}$ E. NOTA

12. Find the number of terminating zeroes at the end of $2011! - 1210!$.

    A. 198   B. 501   C. 1210   D. 2011   E. NOTA

13. Given the function $y = x^2 + 9x + 30$, what is the least possible value of y for integer values of x?

    A. 9   B. $9\dfrac{3}{4}$ C. $9\dfrac{7}{8}$ D. 10   E. NOTA

14. If the point $(x, -4)$ lies on the line joining $(0, 8)$ and $(-4, 0)$, find the value of x.

    A. -2 B. -6 C. 8   D. 6   E. NOTA

15. Amy Li and A. J. McCarron are sitting on a see-saw. Amy weighs 80 pounds and is 5 ft from the center while A. J. weighs 200 pounds, how far away is A. J. from the center if the see-saw is balanced?

    A. 1 ft   B. 12 ft   C. 2 ft   D. 3 ft   E. NOTA

16. If Enrico has a 10% chance of slacking in math on even days, and Mr. Taylor has a 15% chance of yelling at Farhan on even days, what is the probability that on an even-numbered day Enrico slacks in math and Mr. Taylor doesn't yell at Farhan?

    A. $\dfrac{17}{200}$ B. $\dfrac{19}{200}$ C. $\dfrac{23}{200}$ D. $\dfrac{3}{20}$ E. NOTA

17. Find all values of c so that $-4x^2 + 3x + 5c = 0$ has no real solutions.

    A. $c \le -\dfrac{9}{80}$ B. $c > -\dfrac{9}{80}$ C. $c = -\dfrac{9}{80}$ D. $c \ge -\dfrac{9}{80}$ E. NOTA

18. Find the remainder when $x^6 - 6x^5 + 5x^4 - 4x^3 + 3x^2 - 2x - 1$ is divided by $x - 5$.

    A. -348 B. -434 C. -436 D. -347 E. NOTA

19. Find the value of x if $3^{2011} = 3^{2010} + 3^{2010} + x$.

    A. 2010   B. 3^{2010} C. 3   D. 1   E. NOTA

20. If $\begin{aligned} 6i + 7j - 2k &= 3 \\ 3i - 3j - 4k &= 10, \\ 8i + 16j - k &= 17 \end{aligned}$ find $i + j + k$.

    A. -21 B. 10   C. -24 D. -13 E. NOTA

21. If each letter stands for a different digit (1-9), find the two possible values for E.

$$\frac{N \cdot I \cdot N \cdot E \cdot T \cdot E \cdot E \cdot N}{N \cdot I \cdot N \cdot E \cdot T \cdot Y \cdot F \cdot I \cdot V \cdot E} = \frac{O \cdot N \cdot E}{F \cdot I \cdot V \cdot E}$$

   A. 7, 8   B. 6, 7   C. 6, 8   D. 5, 9   E. NOTA

22. Aashka, Botong, Connie, and Daniel are standing on the Cartesian plane. Aashka is standing at (1, 1), Botong is standing at (2, 3), Connie is standing at (6, 2), and Daniel is standing at (5, 3). What is the area of the quadrilateral that they form?

   A. 12   B. 31   C. 15   D. 6   E. NOTA

23. If $f(x) = 4^x$, then $f(x+1) - f(x)$ equals

   A. 4   B. $f(x)$ C. $4f(x)$ D. $2f(x)$ E. NOTA

24. How many ways can a bug get from A to D if it must pass through B and C, and it can only move up and to the right?
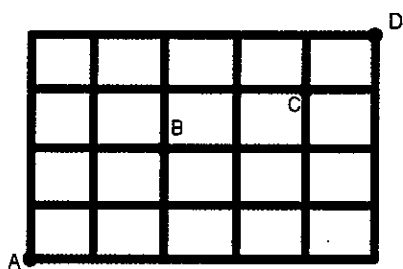
   A. 36   B. 9   C. 18   D. 27   E. NOTA

25. In Country X, there are x number of peeps. 10% of peeps like all three types of pie available in Country X. There are 35% who only like two types of pie. Given that 50% of the population like apple pie, 50% like cherry pie, and 50% like blueberry pie, how many peeps are in Country X if one peep hates all types of pie?

   A. 16   B. 20   C. 24   D. 28   E. NOTA

PLEASE WRITE YOUR NAME, COMPLETE SCHOOL NAME, AND TIE-BREAKER ANSWERS ON THE BACK OF THE SCANTRON FORM. DENOTE EACH TIE-BREAKER AS T1, T2, AND T3.

T1: Once upon a time, there were 5 positive integers: 1, 2, x, y, and z. Their average was 7, their mode was 2, and their median was 2. What is the largest possible value of z?

T2: On Enrico's ranch there are three-legged cows and two-legged cowboys. If there are 348 heads and 913 legs, what is the positive difference between the number of cows and the number of cowboys?

T3: What is the sum of the coefficients in the simplified, expanded form of $(3a - 4b + 3c)^7$?

YOU MAY KEEP THIS COPY OF THE EXAM.