

**2010 Vestavia Hills High School
Mathematics Tournament
Comprehensive Written Examination**

- Find the volume of the solid formed by revolving the region enclosed by the ellipse $9x^2 + 18y^2 = 18$ about the x -axis.
 A. $\frac{4\sqrt{2}}{3}\pi$ B. 2π C. 18π D. $\frac{2\sqrt{2}}{3}\pi$ E. NOTA
- If two of the roots of $x^4 + ax^3 + bx^2 + cx + d = 0$ are $1+i$ and $2-\sqrt{3}$ for all real a, b, c, d , then what is $a+c$?
 A. -16 B. -12 C. 6 D. 4 E. NOTA
- How many ways can Jennifer divide up her 20 dumplings among her 5 friends if each friend must have at least two pieces?
 A. 252 B. 1001 C. 3003 D. 5040 E. NOTA
- Simplify $\sum_{j=0}^k \sum_{a=0}^j a^a$.
 A. $2^{k+2} - 2 - k$ B. $2^{k+3} - 2^k + k$ C. $2^{k+2} - 3 - k$ D. $2^k + 2 - k$ E. NOTA
- Find the area of the triangle with vertices $(0, 0, 0)$, $(5, 7, 4)$, and $(7, 0, 8)$. Answers are in square units.
 A. 37 B. 75 C. 25 D. 42 E. NOTA
- Find the area of the graph bound by the locus of points in which the sum of the distances to the points $(1, 2)$ and $(7, 10)$ is 26.
 A. 130π B. 156π C. 169π D. 260π E. NOTA
- Simplify $\sqrt{\tan^2 x + 8\cos^2 x - 3 + (2\sin x + 2\cos x)(2\sin x - 2\cos x)}$.
 A. $\sin x$ B. $\cos x$ C. $\sec x$ D. $\csc x$ E. NOTA
- If $f(x)$ is the remainder when $x^{2010} - 3x + 4$ is divided by $x^2 - 6x + 8$, find the remainder when $f(8)$ is divided by 16.
 A. 0 B. 1 C. 8 D. 13 E. NOTA
- Suyoung loves the K-Pop group Girls' Generation. His chances of ever meeting them is determined by the function $\frac{1}{2}(1+2x)^{\frac{1}{3\ln x}}$, where x is the numerical quantity of his obsession and $x \geq 9$. If his obsession is infinite, what is the probability that he will meet them?
 A. 1 B. $\frac{1}{6}$ C. $\frac{\sqrt[3]{e}}{2}$ D. $\frac{e}{2}$ E. NOTA
- A circle has a diameter with endpoints at $(1, 0)$ and $(b, 0)$, where $b > 1$. A tangent to the circle has equation $y = \frac{4}{3}x$. Find the value of b .
 A. 4 B. 5 C. 9 D. 11 E. NOTA

11. Find the product of the solutions to $x^{1+\log_3 x} = 729$.
- A. $\frac{1}{3}$

B. 1

C. 3

D. 9

E. NOTA
12. Find the sum of the solutions to $|x+1|+|x-3|=|x+2|$.
- A. -2

B. 0

C. 4

D. 6

E. NOTA
13. Simplify $\sqrt{(111,111,111,111)(1,000,000,000,005)+1}$.
- A. 333,333,333,335

B. 333,333,333,332

C. 333,333,333,334

D. 333,333,333,333

E. NOTA
14. In the xy -plane, consider the L-shaped region bounded by the x - and y - axes with vertices at $(0, 0)$, $(0, 3)$, $(3, 3)$, $(3, 1)$, $(5, 1)$, and $(5, 0)$. Find the slope of the line through the origin that divides the area of this region exactly in half.
- A. $\frac{2}{7}$

B. $\frac{2}{3}$

C. $\frac{3}{4}$

D. $\frac{7}{9}$

E. NOTA
15. What shape is formed by the polar equation $r = 2010 + 2010\sin \theta$?
- A. cardioid

B. circle

C. rose

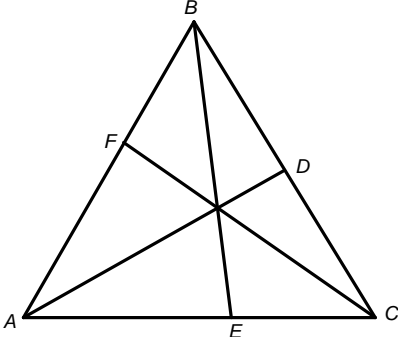
D. hyperbola

E. NOTA
16. Find the length of \overline{BF} , given that $BC = 4$, $AE = 5$, $EC = 3$, $AF = 4$, and \overline{AD} is a median to \overline{BC} .
- A. $\frac{12}{5}$

B. $\frac{8}{3}$

C. $\frac{5}{2}$

D. 3

E. NOTA
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17. A trapezoid is formed from the points on the graph of $y = \frac{2}{x}$ and $y = -\frac{1}{x^3}$, and their respective tangent lines to the y -axis. Find its area as x approaches infinity.
- A. 0

B. $\frac{3}{2}$

C. 3

D. $\frac{5}{2}$

E. NOTA
18. If the solution to $3x^2 + y^2 + z^2 = 2x(y + z)$ is the ordered triple (a, b, c) , find the sum of a , b , and c .
- A. 0

B. 1

C. 2

D. 3

E. NOTA
19. Evaluate for $n = 20102010$: $\left\lfloor \sqrt{n^2 - 16n + 69} \right\rfloor$.
- A. 20102010

B. 20101994

C. 20102002

D. 20102079

E. NOTA
20. How many digits are in x if $\sum_{n=0}^{1005} \binom{2010}{2n} = x$?
- A. 605

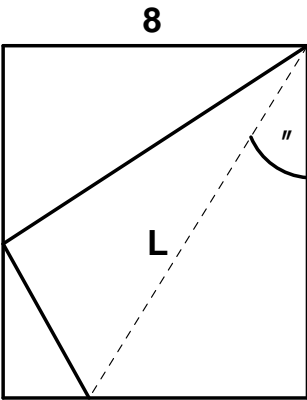
B. 958

C. 604

D. 959

E. NOTA

21. A rectangular piece of paper 8 inches wide is folded as in the diagram so that one corner touches the opposite side. Find the length of the crease L in terms of the angle θ .
- A. $8\sin\theta\sec\theta$ B. $4\csc\theta\sec^2\theta$ C. $4\sec\theta\csc\theta$
- D. $6\sec\theta\csc^2\theta$ E. NOTA



22. Find the value of $i\ln\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)$.
- A. 0 B. π C. $-\frac{\pi}{3}$ D. $\frac{2\pi}{3}$ E. NOTA
23. How many triangles with positive area are there whose vertices are points in the xy -plane and whose coordinates are integers satisfying $1\leq x\leq 4$ and $1\leq y\leq 4$?
- A. 496 B. 560 C. 512 D. 516 E. NOTA
24. In base R_1 , the expanded fraction F_1 is $0.\overline{37}$ and the expanded fraction F_2 is $0.\overline{73}$. In base R_2 , the fraction F_1 is $0.\overline{25}$ while F_2 is $0.\overline{52}$. What is the sum of R_1 and R_2 in base 10?
- A. 22 B. 21 C. 20 D. 19 E. NOTA
25. What are the last two digits of 2^{2010} ?
- A. 24 B. 44 C. 64 D. 94 E. NOTA

PLEASE WRITE YOUR NAME, COMPLETE SCHOOL NAME, AND TIE-BREAKER ANSWERS ON THE BACK OF THE SCANTRON FORM. DENOTE EACH TIE-BREAKER AS T1, T2, AND T3.

- T1. The solutions to $3^{2x}-7\bullet 3^x+10=0$ are $\log_3 A$ and $\log_3 B$, where $A>B$. Find B .
- T2. What x -value on the graph of $y=\sqrt{\ln x}$ is closest to $(4,0)$?
- T3. An ant is walking along the edges of triangle ABC . It starts at vertex A and walks along 10 edges before it stops. What is the probability that the ant stops at vertex A ?

YOU MAY KEEP THIS COPY OF THE EXAM.