2010 Vestavia Hills High School
Mathematics Tournament
Algebra II Written Examination

1. Find the total number of integral divisors of 288.

   A. 10 B. 36 C. 18 D. 20 E. NOTA

2. If the roots of the equation $8x^4 - 6x^3 + 7x^2 - 8x + 4 = 0$ are *a*, *b*, *c*, and *d*, find $bcd + acd + abd + abc$.

   A. 0.75 B. 0.5 C. –1 D. 1 E. NOTA

3. Evaluate $\sum_{x=1}^{10} \log_{10!} x$.

   A. 10! B. 1 C. 0 D. 0.1 E. NOTA

4. A fair six-sided die is cast four times. Find the probability of obtaining exactly one 6 in the four throws.

   A. $\frac{125}{1296}$ B. $\frac{125}{324}$ C. $\frac{161}{216}$ D. $\frac{2}{3}$ E. NOTA

5. If $f\left(x^2+5\right) = x^2 + x + 2$, find $f(x+1)$ for all positive *x*.

   A. 3 B. $2x + \sqrt{x-5}$ C. $x + 1 + \sqrt{x-4}$ D. $2x + 1 + \sqrt{x-4}$ E. NOTA

6. Which of the following is equivalent to $\frac{1-\cos^2 x}{\cos^2 x}$? Assume $\cos^2 x \neq 0$.

   A. $-\tan^2 x$ B. $\tan^2 x$ C. $\sin^2 x$ D. $\cot^2 x$ E. NOTA

7. Find the eigenvalue(s) of $\begin{bmatrix} 3 & -1 \\ 1 & 5 \end{bmatrix}$.

   A. 4 B. 5 C. 4 and 5 D. 3 and 5 E. NOTA

8. Find the coefficient of the third term in the expansion of $\left(3x + 7y\right)^{-1}$.

   A. $\frac{49}{27}$ B. $\frac{27}{49}$ C. $\frac{9}{343}$ D. $\frac{343}{9}$ E. NOTA

9. Find the distance between the center and a focus of $2x^2 + 5y^2 - 4x - 30y + 37 = 0$.

   A. $\sqrt{7}$ B. $\sqrt{3}$ C. 7 D. 3 E. NOTA

10. Given: $a^{\log_3 7} = 27$, $b^{\log_7 11} = 49$, $c^{\log_{11} 25} = \sqrt{11}$. Find $a^{(\log_3 7)^2} + b^{(\log_7 11)^2} + c^{(\log_{11} 25)^2}$.

    A. 3141 B. 2121 C. 48 D. 469 E. NOTA

11. Emily invests \$2634.12 into a bank account that pays 2% interest, compounded continuously. How long, in years, will it take for her investment to triple?

A. $\frac{100\ln 2}{3}$ B. $\frac{100\ln 3}{3}$ C. $50\ln 2$ D. $50\ln 3$ E. NOTA

12. Let $A=\begin{vmatrix} 1 & 0 & \frac{1}{4} \\ 3 & -4 & \frac{3}{4} \\ 8 & -\frac{32}{3} & 2 \end{vmatrix}$ and $B=\log_3 18-\log_3 2$. Find $2A-B^2$.

A. –10 B. –4 C. 0 D. 15 E. NOTA

13. If the equation $x^3-21x^2+8x+36=0$ has roots *a*, *b*, and *c*, find $3abc+ac^2+bc^2+ab^2+a^2b+a^2c+b^2c$.

A. 168 B. 172 C. –168 D. –172 E. NOTA

14. A chinchilla is jumping from tree to tree. Each jump is half as long as the previous jump, except for each fourth jump, which is one-fourth the length of the previous jump. If the first jump has length 1 unit, and the chinchilla is hyperactive and can jump indefinitely in the same pattern, what is the total distance the chinchilla covers?

A. $\frac{58}{31}$ B. $\frac{7}{4}$ C. $\frac{28}{15}$ D. $\frac{62}{33}$ E. NOTA

15. Find the remainder when $x^{100}-2x^{99}+4x^{98}-8x^{97}+2x-1$ is divided by x^2-3x+2.

A. $11x-7$ B. $7x-11$ C. $2x+3$ D. $3x+2$ E. NOTA

16. Simplify $\frac{\sqrt{12+6\sqrt{3}}}{\sqrt{3}+1}$.

A. $\sqrt{3}$ B. $\frac{3\sqrt{6}-3\sqrt{2}}{8}$ C. $\sqrt{6+3\sqrt{3}}$ D. $\frac{\sqrt{6}-\sqrt{2}}{4}$ E. NOTA

17. Evaluate $\left(\sum_{x=1}^{2010} i^x\right)^{20}$ if $i=\sqrt{-1}$.

A. 2010*i* B. $-\left(2^{2010}\right)$ C. –1005*i* D. –1024 E. NOTA

18. If the domain of function $f(x)$ is $\left(-4,-\frac{5}{8}\right]$, what is the domain of $f\left(|x|\right)-4$?

A. $\left(-8,-\frac{37}{8}\right]$ B. $\left[\frac{5}{8},4\right)$ C. $\left(-4,\frac{5}{8}\right]$ D. $\left[\frac{37}{8},8\right)$ E. NOTA

19. If $\vec{a}=\langle 5,-1\rangle$, $\vec{b}=\langle 0,7\rangle$, $\vec{c}=\langle -4,-6\rangle$, and $\vec{d}=\langle -15,37\rangle$, find $5\left(\vec{a}+\vec{b}-2\vec{d}+3\vec{c}\right)\bullet\left(4\vec{b}-5\vec{c}\right)$.

A. –22640 B. –14280 C. –66526000 D. –57362000 E. NOTA

20. According to Mr. René Descartes, the function $r(x) = x^6 - x^5 + 1000x^4 - 900x^3 - 1001x^2 + 93x + 8$ could not possibly have which of the following root combinations?

A. 4 positive, 0 negative, 2 imaginary
B. 2 positive, 0 negative, 4 imaginary
C. 3 positive, 3 negative, 0 imaginary
D. 0 positive, 0 negative, 6 imaginary
E. NOTA

21. Find the value of x such that $AB = \begin{bmatrix} 4 & 0 & -2 \\ 2 & 3 & 6 \\ -1 & 7 & 5 \end{bmatrix}$ and $BA = \begin{bmatrix} 1 & -4 & x \\ -3 & 4 & -1 \\ 6 & -5 & 2 \end{bmatrix}$.

A. $\frac{145}{9}$
B. $\frac{107}{9}$
C. $-\frac{193}{9}$
D. not possible
E. NOTA

22. Evaluate $\frac{1}{2} + \frac{2}{4} + \frac{3}{8} + \frac{4}{16} + \ldots$.

A. 0
B. 1
C. 2
D. ∞
E. NOTA

23. If Manik writes the number 142 in base x, he gets 11A. If he writes 142 in base y, he gets BA. What is the sum of the reciprocals of x and y?

A. $\frac{24}{143}$
B. $\frac{25}{156}$
C. $\frac{23}{132}$
D. $\frac{27}{182}$
E. NOTA

24. A circle has a diameter equal to the product of the solutions to $|x-3| - |3x-12| + 7 = 0$. Find the area of the circle.

A. 16π
B. 8π
C. 4π
D. 9π
E. NOTA

25. Evaluate using $\pi = 3.14$: $\lceil \pi \rceil + \lfloor \pi \rfloor + \lceil -\pi \rceil + \{\pi\} + \{-\pi\}$.

A. 5
B. 4.28
C. 4
D. 3.28
E. NOTA

PLEASE WRITE YOUR NAME, COMPLETE SCHOOL NAME, AND TIE-BREAKER ANSWERS ON THE BACK OF THE SCANTRON FORM. DENOTE EACH TIE-BREAKER AS T1, T2, AND T3.

T1. Gene and Kyle agree to meet at Mr. Chen's Authentic Chinese Cooking, but they fail to agree upon a meeting time. Kyle will arrive at Mr. Chen's sometime between 12 p.m. and 6 p.m. and will stay for an hour before leaving. Gene will also arrive at a random time between 12 p.m. and 6 p.m., but he is willing to wait for two hours before leaving. What is the probability that both boys will be in the restaurant at the same time?

T2. Find the number of digits in the expansion of 2^{100}.

T3. Find the number of positive integers that are relatively prime to 244.

YOU MAY KEEP THIS COPY OF THE EXAM.