

# 2009 Vestavia Hills High School Mathematics Tournament Geometry Written Examination

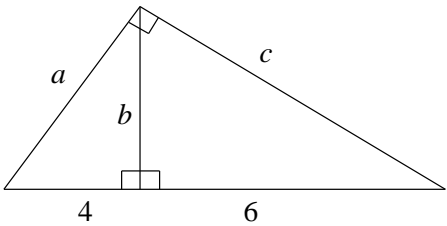
1. Find  $a + b + c$  using the diagram to the right.

- A.  $\frac{1+\sqrt{5}}{2}$

B.  $3\sqrt{2} + 4\sqrt{2} + 5\sqrt{6}$
- C. 24

D.  $2\sqrt{10} + 2\sqrt{6} + 2\sqrt{15}$

E. NOTA



2. Find the number of vertices in a dodecahedron.

- A. 14

B. 20

C. 22

D. 26

E. NOTA

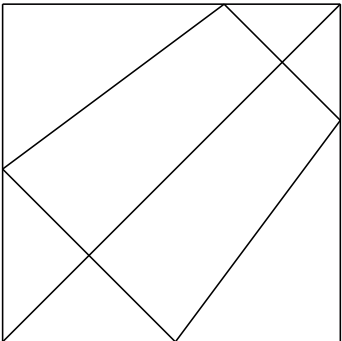
3. Each vertex of an isosceles trapezoid is on a different side of a square so that one of the square’s diagonals is an axis of symmetry of the trapezoid. If the area of the trapezoid is 63 units<sup>2</sup>, and the distance between the bases of the trapezoid is 7 units, what is the area of the square? All answers are in square units.

- A. 154

B. 130

C. 172
- D. 128

E. NOTA



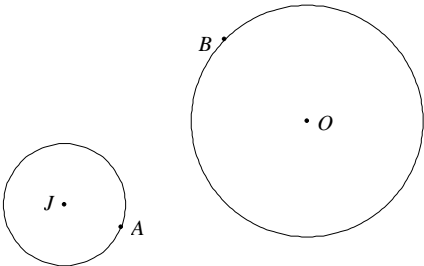
4. Given  $\odot J$  with radius 4 and  $\odot O$  with radius 6 units, find the shortest distance between points  $A$  and  $B$  if points  $J$  and  $O$  are 12 units apart and  $\overline{AB}$  is an internal tangent. All answers are in units.

- A.  $2\sqrt{11}$

B.  $4\sqrt{5}$

C.  $4\sqrt{11}$
- D.  $3\sqrt{5}$

E. NOTA



5. A point  $P(x, y)$  moves in such a way that its distance from  $C(2, -1)$  is always 5. Find the equation of its locus.

- A.  $x^2 + y^2 - 4x + 2y = 0$

B.  $x^2 + y^2 - 4x + 2y - 20 = 0$

C.  $x^2 + y^2 + 4x - 2y = 0$
- D.  $x^2 + y^2 + 4x - 2y - 20 = 0$

E. NOTA

6. The lengths of the legs of a certain right triangle are 18 units and 63 units. What is the area of the circle tangent to both legs of the triangle and has its center on the hypotenuse? All answers are in square units.

- A.  $169\pi$

B.  $196\pi$

C.  $144\pi$

D.  $256\pi$

E. NOTA

7. There are seven Democrats and eight Republicans sitting at a round table with fifteen seats. How many ways can they be seated if the Republicans must sit together and the Democrats must sit together?

- A.  $7!7!$

B.  $\frac{7!7!}{2}$

C.  $7!8!$

D.  $\frac{7!8!}{2}$

E. NOTA

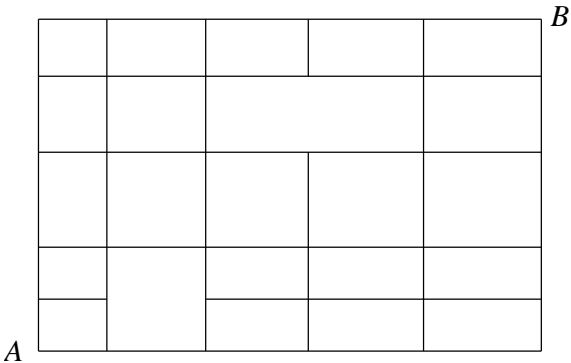
8. Traveling only up and to the right along the given segments, how many ways are there to get from  $A$  to  $B$ ?

- A. 100

B. 121

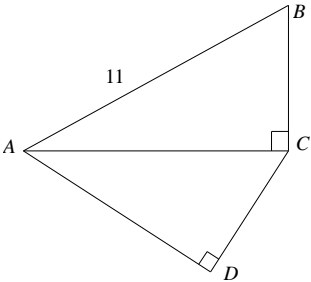
C. 132
- D. 140

E. NOTA



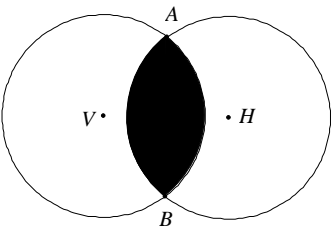
9. In quadrilateral  $ABCD$ ,  $\overline{AC} \perp \overline{BC}$  and  $\overline{AD} \perp \overline{CD}$ , as shown.  
If  $AB = 11$  units, what is the sum of the squares of the lengths of the sides of this quadrilateral?

A. 22                      B. 132                      C. 242                      D. 121                      E. NOTA



10. Circles  $V$  and  $H$  are congruent and intersect at points  $A$  and  $B$ .  
If each circle has radius 6 units, and  $AB = 6$  units, what is the area of the shaded region?

A.  $12\pi - 18\sqrt{3}$                       B.  $18\pi - 12\sqrt{3}$                       C.  $36\pi$   
D.  $9\sqrt{3} - 6$                       E. NOTA



11. A cylinder of radius  $\frac{3}{2}$  units has a height of  $8\pi$  units. If an ant starts at the bottom and moves around the cylinder exactly five times before reaching the top, how far has it traveled? All answers are in units.

A.  $\sqrt{73}\pi$                       B.  $5\pi$                       C.  $\frac{\sqrt{481}}{2}\pi$                       D.  $17\pi$                       E. NOTA

12. In the land of Hullahullawhaboo, each Haperdingle owns a certain number of Quinoxafams.  
A Quinoxafam is a silver ball used for currency; Haperdingles under age 14 are children and they have 2 Quinoxafams with radius 2 feet. Haperdingles age 14 and above have 3 Quinoxafams of radius 3 feet.  
If the ages of seven Haperdingles are 7, 8, 9, 11, 14, 15, and 15, find the total surface area of their currency in square inches.

A.  $452\pi$                       B.  $128\pi$                       C.  $324\pi$                       D.  $3120\pi$                       E. NOTA

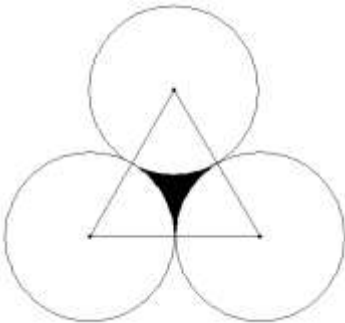
13. The following statements were made on the same day:
- It was Monday yesterday.
  - Today is Thursday.
  - The day after tomorrow will be Friday.
  - Tomorrow will be Saturday.
  - The day before yesterday was Tuesday.

Given that the number of correct statements above uniquely determines the day of the week the statements were made, what day of the week were the statements made?

A. Monday                      B. Tuesday                      C. Wednesday                      D. Thursday                      E. NOTA

14. Find the area of the shaded region. The circles are mutually tangent and each has a radius of 5 units. All answers are in square units.

A.  $\frac{25}{3}\sqrt{3}$                       B.  $30\sqrt{3} + 10\pi$                       C.  $30\sqrt{3} - 10\pi$   
D.  $\frac{50\sqrt{3} - 25\pi}{2}$                       E. NOTA



15. Find the shortest distance between the point  $(-3, 8)$  and the line determined by  $(6, 10)$  and  $(-2, -5)$ .

A. 7                      B. 8                      C.  $4\sqrt{3}$                       D.  $5\sqrt{3}$                       E. NOTA

16. The length of the internal tangent between two circles is 39 units. The length of the external tangent between the same circles is 60 units. If the distance between their centers is 65 units, find the product of their radii.

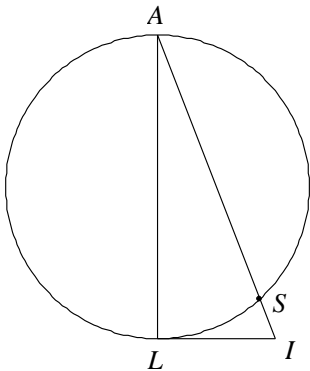
- A.  $\frac{2739}{4}$ 
B.  $\frac{2079}{4}$ 
C. 1300
D.  $\frac{1679}{4}$ 
E. NOTA

17. Sunny High School offers three history courses: History of Shi, History of Li, and History of Xie. Fifty students take History of Shi, 40 take History of Li, 30 take History of Xie, 10 take Shi and Li, 5 take Li and Xie, 8 take Shi and Xie, and 2 take all three classes. If 100 students attend the school, how many take none of the classes?

- A. 1
B. 12
C. 13
D. 20
E. NOTA

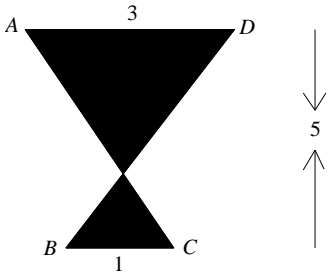
18. Find  $AL$ , the diameter of the circle shown, if  $IS = \sqrt{3}$  and  $m\angle LAI = 30^\circ$ .

- A.  $3\sqrt{3}$ 
B. 6
C.  $4\sqrt{3}$ 
D. 8
E. NOTA



19. Given:  $\overline{AD} \parallel \overline{BC}$ ,  $BC = 1$ ,  $AD = 3$ , distance between  $\overline{AD}$  and  $\overline{BC}$  is 5. Find the sum of the areas of the triangles.

- A. 6
B. 6.25
C. 6.5
D. 6.75
E. NOTA



20. Find the number of diagonals in a regular 18-gon.

- A. 288
B. 144
C. 270
D. 135
E. NOTA

21. In  $\triangle ABT$ ,  $\overline{AZ}$  bisects  $\angle A$  and intersects  $\overline{BT}$  at  $Z$ . If  $AB = 10$ ,  $AT = 12$ , and  $BZ = 5$ , find  $ZT$ .

- A. 6
B.  $\sqrt{69}$ 
C. 17
D. 24
E. NOTA

22. Ricky bought electric blue athletic shoes to motivate himself to run the route he planned out. In his plan, he will run west from his house for 30 minutes at a rate of 10 blocks per hour, then turn right for 45 minutes at 20 blocks per hour, then turn right again for 20 minutes at 15 blocks per hour, then turn right and return home in 10 minutes. Find the positive difference between the area Ricky’s path encloses and the number of minutes it takes him to run the route. Assume that all turns are right angles and all blocks are the same length.

- A. 75
B. 105
C. 20
D. 30
E. NOTA

23. Evaluate  $\frac{\sin X}{\cos B}$  when  $X = 60^\circ$  and  $B = 30^\circ$ .

- A. 1
B.  $\sqrt{3}$ 
C.  $\frac{\sqrt{3}}{3}$ 
D.  $\frac{\sqrt{2}}{2}$ 
E. NOTA

24. Julie has an ice cream cone full of ice cream, and the ice cream protrudes out of the cone as a hemisphere. Both the ice cream cone and the ice cream hemisphere have a radius of two inches. Julie eats her ice cream until she gets a brain freeze, when the ice cream reaches only half its original height inside the cone. How much of the ice cream (volume) did Julie consume, if the height of the cone is 6 inches? All answers are in cubic inches.
- A.  $12\pi$                       B.  $\frac{40}{3}\pi$                       C.  $\frac{37}{3}\pi$                       D.  $\frac{32}{3}\pi$                       E. NOTA
25. Find the distance between the centroid of  $\triangle ABC$  and point  $C$  if  $A(-2, 6)$ ,  $B(5, 1)$ , and  $C(3, 2)$ .
- A.  $5\sqrt{2}$                       B.  $5\sqrt{5}$                       C.  $2\sqrt{2}$                       D.  $\sqrt{2}$                       E. NOTA

**PLEASE WRITE YOUR NAME, COMPLETE SCHOOL NAME, AND TIE-BREAKER ANSWERS ON THE BACK OF THE SCANTRON FORM. DENOTE EACH TIE-BREAKER AS T1, T2, AND T3.**

- TB1. Find the number of distinct arrangements of VESTAVIA divided by the number of distinct arrangements of REBELS.
- TB2. Beginning at 5:00 a.m., how many hours (or what part of an hour) must elapse before the hour and the minute hand of a clock form a right angle?
- TB3.  $\triangle ABC$  has medians  $\overline{AE}$ ,  $\overline{CD}$ , and  $\overline{BF}$ . If their lengths are 5 units, 7 units, and 10 units, respectively, find the area of  $\triangle ABC$ .

**YOU MAY KEEP THIS COPY OF THE EXAM.**