## 2009 Vestavia Hills High School Mathematics Tournament Algebra II Written Examination

1. Evaluate  $2 + \sqrt{3 + \sqrt{3 + \sqrt{3 + \dots}}}$ . A.  $\frac{2+\sqrt{13}}{2}$  B. 5 C.  $\frac{5-\sqrt{3}}{2}$  D.  $\frac{5+\sqrt{13}}{2}$ E. NOTA 2. Find  $\left(\frac{\sqrt{3}}{2} - \frac{1}{2}i\right)^{2009}$ . A.  $-\frac{\sqrt{3}}{2} - \frac{1}{2}i$  B.  $\frac{\sqrt{3}}{2} - \frac{1}{2}i$  C.  $\frac{1}{2} + \frac{\sqrt{3}}{2}i$  D.  $-\frac{\sqrt{3}}{2} + \frac{1}{2}i$ E. NOTA 3. Kangaroo Kyle makes semicircular arcs when he jumps. He gets tired fast so his jump size decreases in a geometric manner. The heights of his first three bounces are 25, 20, and 16 feet. If this pattern continues, how far will he travel in the air before coming to rest? Α. 150π B. 250π C. 125π D. 100π E. NOTA 4. Find the sum of the solutions for *x*:  $(x^2 - 7x + 11)^{(x^2 - 11x + 30)} = 1$ . A. 11 B. 13 C. 18 D. 25 E. NOTA 5. Solve for x:  $\frac{x}{x-1} < \frac{1}{x+3}$ . B.  $(-\infty, -1)$  C.  $(-\infty, -3) \cup (1, \infty)$ A.  $(-3, -1) \cup (-1, 1)$ D.Ø E. NOTA 6. The product of two positive three-digit palindromes is 436995. What is their sum? A. 1332 B. 1130 C. 1231 D. 1433 E. NOTA 7. What is the maximum height that an 8-foot-wide truck can be in order to pass through a parabolic tunnel that is 18 feet high and 12 feet wide? All answers are in feet. A. 12 B. 14 C. 10 D. 8 E. NOTA 8. If  $\sin A = \frac{4}{5}$  and  $\sin B = -\frac{12}{13}$ , where  $0 < A < \frac{\pi}{2}$  and  $\frac{3}{2}\pi < B < 2\pi$ , find  $\sin(A+B)$ . A.  $-\frac{8}{9}$  B.  $-\frac{18}{65}$  C.  $-\frac{16}{25}$  D.  $\frac{24}{25}$ E. NOTA 9. Suyoung goes on a date with all nine members of Girls' Generation (Taeyeon, Yoona, Jessica, Tiffany, Hyoyeon, Yuri, Sooyoung, Seohyun, and Sunny) and they all sit at a circular table with equally spaced chairs. If Suyoung must sit between Taeyeon and Jessica, how many different ways can they sit together? A. 10080 B. 362880 C. 5040 D. 80640 E. NOTA 10. Find the product of the roots of the equation  $10^{\log_x 10} = 10x^2$ . A.  $\frac{1}{10}$  B.  $\frac{\sqrt{10}}{10}$ C. 1 D. -2 E. NOTA

- 11. If -2 and 1-i are roots of a cubic equation with integral coefficients, find the sum of the coefficients of the standard form of the equation, if the coefficients are relatively prime and the leading coefficient is positive.
  - A. 0 B. 1 C. 2 D. 3 E. NOTA
- 12. If  $x + \frac{1}{x} = 9$ , what is  $x^3 + \frac{1}{x^3}$ ? A. 729 B. 720 C. 702 D. 700 E. NOTA 13. Find the sum of the squares of the roots of  $\begin{vmatrix} x & 0 & 1 \\ 3 & x & -2 \\ -4 & 6 & 10 \end{vmatrix} = 10$ .
  - A.  $\frac{24}{25}$  B. 56 C.  $-\frac{24}{13}$  D.  $-\frac{8}{5}$  E. NOTA
- 14. Find the sum of the solutions of  $|x+1| = -\frac{x}{7}$ .
  - A.  $\frac{7}{8}$  B.  $-\frac{7}{8}$  C.  $-\frac{7}{6}$  D. 0 E. NOTA
- 15. Find  $\sum_{x=7}^{\infty} \frac{a}{x^2 7x + 12}$ , where *a* is the remainder when  $x^{2008} 2$  is divided by x i, where *i* is the imaginary unit.
  - A.  $-\frac{1}{2}$  B.  $-\frac{1}{3}$  C.  $\frac{1}{7}$  D. sum is divergent E. NOTA
- 16. Forrest can dust the math team trophy case in 40 minutes, Brandon can dust the trophy case in 50 minutes, Zhen can dust the trophy case in 60 minutes, and Andrew can dust the trophy case in four hours. How long, in hours, will it take the boys to dust the trophy case, working together?
  - A.  $\frac{1}{4}$  B.  $\frac{59}{240}$  C.  $\frac{15}{67}$  D.  $\frac{20}{79}$  E. NOTA
- 17. What is the volume of the frustum formed by rotating y = |x| about the x-axis, where  $3 \le x \le 6$ ?
  - A.  $27\pi$  B.  $63\pi$  C.  $64\pi$  D.  $72\pi$  E. NOTA
- 18. What is the area of the shape bounded by 9y+3x=27, 3x-4y=-12, and the *x*-axis?
  - A. 19.5 B. 21.5 C. 22 D. 23 E. NOTA
- 19. Find  $\frac{A\sqrt{B}}{D}$  if:
  - A = the remainder when  $5x^4 \frac{3}{2}x^2 + x 7$  is divided by x 2; B = the radius of the circle  $x^2 + y^2 - 12x + 4y + 36 = 0$ ; and D =  $(\log_9 36)(\log_2 3)(\log_1 10^6)(\log_6 4)$ .
  - A.  $\frac{69\sqrt{2}}{8}$  B.  $\frac{69\sqrt{3}}{7}$  C.  $\frac{23\sqrt{2}}{4}$  D. 12 E. NOTA

- 20. Find a+b+c if (a, b, c) is the ordered triple of integers for which  $x = \frac{3}{\sqrt[3]{7}-2}$  is a solution to  $x^3 + ax^2 + bx + c = 0$ .
  - A.  $5-\sqrt{7}$  B. 91 C. 117 D. 82 E. NOTA
- 21. Find the fifth term in the expansion of  $(10x+6y)^8$ .
  - A.  $608,300,000x^3y^5$ B.  $690,300,000x^5y^3$ C.  $907,200,000x^4y^4$ D.  $5,300,000x^3y^5$ E. NOTA
- 22. Daniel and Soojin are supposed to meet for lunch at McDonald's. However, neither of them remembers the time to meet, only that it is between 12 noon and 2:00 p.m. If each of them will only wait for 30 minutes before leaving, what is the probability that they meet?
- A.  $\frac{9}{16}$  B.  $\frac{7}{16}$  C.  $\frac{1}{2}$  D.  $\frac{3}{8}$  E. NOTA 23. Evaluate  $\sum_{i=1}^{\infty} \frac{2i}{3^{i+1}}$ .
  - A.  $\frac{1}{3}$  B.  $\frac{1}{2}$  C. 1 D.  $\frac{2}{3}$  E. NOTA
- 24. Jackie Chan wants to build a rectangular pen to pen up the bad guys. He has 20 feet of fencing to use, but he wants one of the "walls" to be a wall in his basement. The other three sides will be made from the fencing. What is the maximum area of the pen that can be constructed? All answers are in square feet.
  - A. 50 B. 37.5 C. 100 D.  $\frac{400}{9}$  E. NOTA
- 25. Starting at point *A*, Jerry rows one mile upstream to point *B*, when he drops his soccer ball. However, Jerry doesn't realize the loss of the ball. After 10 minutes, he realizes the ball is missing and immediately turns and rows downstream. At point *A*, he overtakes his ball, which has been carried downstream by the current. If Jerry rows at a constant speed relative to the water, what is the rate (in miles per hour) of the current?
  - A. 1 B. 3 C. 5 D. 6 E. NOTA

## PLEASE WRITE YOUR NAME, COMPLETE SCHOOL NAME, AND TIE-BREAKER ANSWERS ON THE BACK OF THE SCANTRON FORM. DENOTE EACH TIE-BREAKER AS T1, T2, AND T3.

TB1. What is the range of the function  $f(x) = \sqrt{x-7}$ ?

TB2. What is the maximum value of 10x + 6y if  $x \ge 0$ ,  $y \ge 0$ ,  $y \le -\frac{1}{2}x + 4$  and  $y \ge -x + 2$ ?

TB3. Evaluate  $\sum_{n=0}^{\infty} \frac{2009}{n!}$ .

## YOU MAY KEEP THIS COPY OF THE EXAM.