

1. In a family of four kids, if at least three of the kids are boys, then what is the probability the oldest child is a girl?

- A. $\frac{1}{8}$ B. $\frac{1}{5}$ C. $\frac{1}{4}$ D. $\frac{1}{2}$ E. NOTA

2. Evaluate $\sec\left(\tan^{-1}\frac{a}{b}\right)$.

- A. $\frac{\sqrt{a^2+b^2}}{a}$
 B. $\frac{\sqrt{a^2+b^2}}{b}$
 C. $\frac{a\sqrt{a^2+b^2}}{a^2+b^2}$
 D. $\frac{b\sqrt{a^2+b^2}}{a^2+b^2}$
 E. NOTA

3. A traveling salesman arrives at the front of a castle with four gates and a sign posted on each gate. The guard at the front of the castle tells the salesman that behind one gate is an eager crowd waiting to buy his wares. Behind the remaining three gates are vicious dogs waiting to bite the seat out of his pants. The sign of the gate behind which waits the crowd is truthful, but the signs on the gates behind which wait the dogs are false. Which gate is the gate behind which the crowd waits?

<p><u>GATE 1</u></p> <p>Gate 4 is truthful.</p>	<p><u>GATE 2</u></p> <p>There are dogs behind this gate and Gate 3.</p>
<p><u>GATE 3</u></p> <p>There are dogs behind Gates 1 and 2.</p>	<p><u>GATE 4</u></p> <p>The crowd is behind this gate.</p>

- A. Gate 1
 B. Gate 2
 C. Gate 3
 D. Gate 4
 E. Cannot be Determined

4. Given the function $f(x) = 6\sqrt{5}\sin\left(11x - \frac{\pi}{3}\right) + 22$, what is the product of the horizontal and vertical shifts of the curve?

- A. $66\sqrt{5}$ B. 242 C. $\frac{2\pi}{11}$ D. $\frac{2\pi}{3}$ E. NOTA

5. Find $\log_a b$ given the following.

$$a = \left((\tan 15^\circ) + \sqrt{3} \right)^5$$

$$b = 4(\cos 116^\circ \cos 244^\circ + \cos 23^\circ \sin 67^\circ + \cos 67^\circ \sin 23^\circ - \sin 116^\circ \sin 244^\circ)$$

- A. $\frac{\pi}{2}$ B. $\frac{1}{2}$ C. $\frac{3}{4}$ D. $\frac{3}{5}$ E. NOTA

6. Find $\langle 9, -5, 4 \rangle \times \langle 7, 6, -3 \rangle$.

- A. $\langle -9, 55, 89 \rangle$
 B. $\langle -9, -55, 89 \rangle$
 C. $\langle 63, -30, -12 \rangle$
 D. $\langle 63, 30, 12 \rangle$
 E. NOTA

7. Find $A - B$.

$$A = \{ \text{domain of } e^{\ln x} \}$$

$$B = \{ \text{domain of } \ln e^x \}$$

- A. $(-\infty, 0]$ B. $(-\infty, 0)$ C. $\{0\}$ D. $\{ \}$ E. NOTA

8. If $\tan A = -\frac{12}{5}$ and $\frac{3\pi}{2} < A < 2\pi$, find $\cos 2A + \sin 2A$.

- A. $\frac{49}{169}$ B. $\frac{289}{169}$ C. $-\frac{49}{169}$ D. $-\frac{239}{169}$ E. NOTA

9. Rewrite the following polar equation as rectangular in standard form.

$$\frac{13\sqrt{2}}{2} = r \cos\left(\theta - \frac{\pi}{4}\right)$$

- A. $x + y = \frac{13}{2}$
 B. $x + y = 13$
 C. $2x + y = 13$
 D. $2x + y = \frac{13}{2}$
 E. NOTA

10. Evaluate: $\frac{\sin^2 75^\circ + \csc^2 75^\circ + \cos^2 75^\circ + \sec^2 75^\circ - \tan^2 75^\circ - \cot^2 75^\circ}{1 + \sum_{n=1}^{\infty} \frac{n}{2^n}}$.

- A. 0.5 B. 1 C. 2 D. 0 E. NOTA

11. Find $A + B$ given the following.

A = the number of petals on the curve $r = 3\sin 6\theta$
 B = the length of each petal on the curve $r = 4\cos 7\theta$.

- A. 7 B. 10 C. 16 D. 17 E. NOTA

12. A right triangle has legs a and b and hypotenuse c . Let a be the eccentricity of the ellipse with equation $16x^2 + 9y^2 - 64x + 54y = -1$ and let b be the positive root of the curve $y = x^3 + 2x^2 - x - 2$. Find the hypotenuse c .

- A. $\frac{\sqrt{11}}{4}$ B. $\frac{\sqrt{23}}{4}$ C. $\frac{\sqrt{7}}{4}$ D. $\sqrt{3}$ E. NOTA

13. Find the tens digit of 21^{705} .

- A. 0 B. 2 C. 4 D. 6 E. NOTA

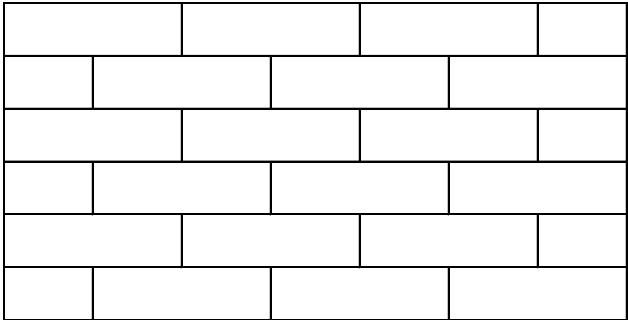
14. Simplify the following: $(\sec^4 \alpha - 2\sec^2 \alpha)(\sec 2\alpha)$.

- A. $\sec^4 \alpha$ B. $\tan^4 \alpha$ C. $-\sec^4 \alpha$ D. $-\tan^4 \alpha$ E. NOTA

15. Simplify $(\log_{125} e)(\log_{81} 25)(\ln 4)(\log_2 9)$.

- A. $\frac{2}{3}$ B. $\frac{3}{2}$ C. e D. $\ln 3$ E. NOTA

16. In how many ways can you move from the bottom left of the diagram below to the upper right of the diagram if you can only move up or right along the lines of the diagram?



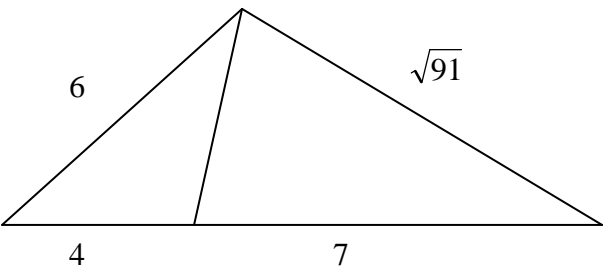
- A. 63 B. 68 C. 79 D. 87 E. NOTA

17. What is the tangent of the angle formed by the hands of a clock at 11:30?

- A. $-\sqrt{3} - 2$ B. $\sqrt{3} - 2$ C. $\sqrt{3} + 2$ D. $-\sqrt{3} + 2$ E. NOTA

18. A tiger is tied to one corner of a regular pentagon with 10 meter side lengths by a 12 meter leash. Find the area around the pentagon in which the tiger may roam.
- A. $125\pi - 30$
 B. $125\pi - 50$
 C. $\frac{512\pi}{5}$
 D. $\frac{504\pi}{5}$
 E. NOTA

19. Find the altitude to the side of length 11 of the triangle below. The figure is not drawn to scale.



- A. 5 B. $\sqrt{29}$ C. $\frac{9}{2}$ D. $3\sqrt{3}$ E. NOTA

20. Let $f(x)$ be a 6th degree function with leading coefficient 1, such that $f(1)=1$, $f(2)=2$, $f(3)=3$, $f(4)=4$, $f(5)=5$, and $f(6)=6$, and $f(10)=60490$. What is $f(7)$?
- A. 0 B. 7 C. 720 D. 727 E. NOTA

21. Find the area of the circle inscribed in the triangle formed by the centroids of $\triangle ABC$, $\triangle CDE$, and $\triangle EFA$ given the points below.

$A(-4, 5)$	$D(7,13)$
$B(-2, 7)$	$E(5,8)$
$C(-9, 0)$	$F(-4,-25)$

- A. $\frac{20\pi}{3}$ B. $\frac{25\pi}{4}$ C. 4π D. 5π E. NOTA

22. Solve the following ($\lfloor x \rfloor$ is the greatest integer less than or equal to x).

$$\sum_{n=1}^{1024} \lfloor \log_2 n \rfloor$$

- A. 8204 B. 8192 C. 9218 D. 1024 E. NOTA

23. Rob, Robby, and Roberta play a game with a fair, single six-sided die. Rob rolls first and wins if he rolls a 1. If he does not roll a 1, then Robby rolls. He wins if he rolls a 1 or 2. If he doesn't win, then Roberta rolls. Roberta wins if she rolls a 1, 2, or 3. If Roberta does not win, then the game starts over. What is the absolute value of the difference between Robby and Roberta's chances of winning?

- A. 0 B. $\frac{1}{9}$ C. $\frac{1}{6}$ D. $\frac{1}{36}$ E. NOTA

24. Let (P, Q) be the ordered pair that defines the point on the coordinate plane halfway between the two horizontal asymptotes on the curve $y = \frac{45}{3 + 12e^{-0.71x}}$. Find the exact value of P .

- A. $\frac{45}{2}$ B. $\frac{15}{2}$ C. $\frac{2\ln 2}{71}$ D. $\frac{200\ln 2}{71}$ E. NOTA

25. A toddler has built a figure with blocks. The figure has four columns with two blocks in each column. If the toddler is only strong enough to move one block at a time and can only move the top block of each column, in how many ways can the toddler move all of the blocks back into the basket in which they belong?

- A. 1260 B. 2520 C. 4320 D. 7560 E. NOTA

TB1 There are 81 balls on the table. All of the balls weigh the same except for one ball, which weighs more. Also on the table is a balance. What is the least number of times you need to use the balance to accurately determine and guarantee the heaviest ball?

TB2 Simplify $\left| (1 + i)^{10} - 6(1 - 2i)^2 \right|$.

TB3 A certain bacteria reproduces by a process known as binary fission. In this type of reproduction, a bacterium divides forming two bacteria. These bacteria then divide into two bacteria and the process continues. Under ideal conditions, the binary fission occurs every 20 minutes. What is the value of the growth constant k that can be used to model binary fission under these conditions?