2008 Hoover HS Math Tournament Comprehensive Ciphering

Practice: Find the sum of the positive integral factors of 2008. 3780

1.1 If $f(x) = Ax^2 + Bx + C$ has a double root, then $\sqrt{\left(\frac{B}{A}\right)\left(\frac{B}{C}\right)} = ?$ 2

1.2 What shape is the graph given by the polar equation $r = \frac{13}{7 - 3\cos\theta}$? ellipse

1.3 Find the largest prime in the factorization of $3^{15} - 243$. 61

1.4 If $f(x) = \frac{x+4}{x-3}$ and $g(x+2) = 6x+3$, find $f(g(g(3)))$. $\frac{7}{6}$

1.5 Find the smallest positive integer c such that $\frac{2}{3}c$ and $\frac{3}{2}c$ are both perfect squares. 6

2.1 When the denominator of the fraction $\frac{3-\sqrt{5}+\sqrt{14}}{3+\sqrt{5}+\sqrt{14}}$ is rationalized and the fraction is simplified, what is the denominator's numerical value? 3

2.2 Find the largest real solution to the equation $3x^4 + 14x^3 + 23x^2 + 16x + 4 = 0$. $-\frac{2}{3}$

2.3 In how many zeros does $2008!$ end when written in base 45? 500

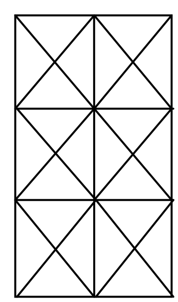
2.4 What is the entry in the second row and second column of the inverse of matrix $A = \begin{bmatrix} 7 & 0 & -9 \\ 2 & 5 & -1 \\ -3 & 8 & 1 \end{bmatrix}$? $\frac{5}{47}$

2.5 Find the sum of the positive integer values of n that satisfy the equation $n^2(n-1)^2 \ldots (2)^2(1)^2 - 144n(n-1)\ldots(2)(1) + 2880 = 0$. 9

3.1 Simplify: $\sqrt{\sum_{i=0}^{2}\left(i+\sqrt{2}\right)^2}$ $3+\sqrt{2}$

3.2 A hyperbola has foci at the points $(1,4)$ and $(1,-2)$ and asymptotes with slopes of $\pm\sqrt{2}$. If the equation of the hyperbola is given in the form $a(y-b)^2 - c(x-d)^2 = 1$, with a, b, c, and d real, find $(a+c)(b+d)$. 1

B



A

3.3 In how many ways can Ms. Campbell get from point A to point B by moving only to the right, up, 45° down and to the right, or 45° up and to the right? Ms. Campbell cannot stop at the intersection in the center of any of the 6 squares. 63

3.4 Find the infinite sum: $1 + \frac{5}{6} + \frac{5}{9} + \frac{17}{54} + \frac{13}{81} + \ldots + \frac{n^2+1}{2(3)^{n-1}} + \ldots$ 3

3.5 A convex equiangular hexagon has consecutive sides of length x, 4, 6, and y. Find the distance between the sides of length x and y. $5\sqrt{3}$

4.1 Find the smallest positive integer that leaves a remainder of $n-1$ when divided by n for each integer n satisfying $5 \le n \le 15$. 360359

4.2 Find the distance between the plane $3x - 2y + 4z = 8$ and the point $(2,-1,3)$. $\frac{12\sqrt{29}}{29}$

4.3 Points A, B, and C are the vertices of an equilateral triangle with point C on a circle and side $\overline{AB}$ tangent to the same circle. If the radius of that circle is 3, then the area of the region inside the intersection of the triangle and the circle is given in the form $a\sqrt{3}+b\pi$, where a and b are real numbers. Find $\frac{a}{b}$. $\frac{3}{2}$

4.4 How many ordered quadruples (x, y, z, w) of nonnegative integers satisfy $x+y+z+w=17$? 1140

4.5 Find the equation of the directrix with larger x-intercept of the hyperbola with equation $5x^2-4y^2+30x+8y+21=0$. $x=-\frac{5}{3}$

E.1 The sum $\sum_{k=1}^{n}\left(k^2-k\right)$ is equal to an^3+bn^2+cn+d. Find $\frac{a+b}{c+d}$. -1

E.2 Find the sum of the solutions to the equation $\sin 3x \cos 3x=\frac{1}{4}$ on the interval $[0,\pi)$. $\frac{5\pi}{2}$