2008 Hoover High School Mathematics Tournament
Algebra 2 Written Test

1. What is the horizontal asymptote of the function $y = \dfrac{-8x^5 - 7x^4 + 12x^3 - 8}{2x^5 + 3x^4 + x^3 - x^2 - 6x}$

A) $y = 0$ B) $y = 4$ C) $x = 4$ D) $y = -4$ E) NOTA

2. Will has a really,-really,-ridiculous-superball (RRRSB) which rebounds to 7 times its original height. From how high, in inches, should Will drop the RRRSB so that after one bounce, it rebounds to the same height as Will? Will is 77 inches tall.

A) 11 B) 7 C) 77 D) the RRRSB will never reach 77 inches exactly E) NOTA

3. What is the smallest positive integer n such that the sum of the first n positive integers is divisible by 24?

A) 24 B) 9 C) 23 D) 14 E) NOTA

4. The class average on the conic sections test, as reported by STI, was a 74. However, Daniel's score was saved by STI as a 74 when it was actually an 84, Farah's score was saved by STI as a 71 when it was actually an 89, and Victor's score was also saved by STI as a 71 when it was actually a 99. This made the actual class average on the conic sections test a 78. How many people are in the class?

A) 3 B) 12 C) 13 D) 14 E) NOTA

5. A sequence $\{a_n\}$ with $a_1 = 2$ and $a_2 = 6$ is defined in the following way: for each term beyond the second, $a_n = $ the sum of the digits of the sum of the squares of a_{n-1} and a_{n-2}. What is the value of a_{2008} ?

A) 1 B) 3 C) 5 D) 8 E) NOTA

6. Since he is bored, Justin is trying to figure out what scores are possible on this test with the current scoring system (+4 points for a correct answer, +0 for a question left blank, −1 for an incorrect answer). Since the possible scores can range from −25 to 100, Justin is curious as to what is the lowest possible score one cannot make. What is the lowest possible score that is unattainable under this scoring system?

A) −14 B) 89 C) 69 D) 25 E) NOTA

7. Hizny is a math whiz but also somewhat paranoid. To prevent anyone who sees his locker combination from getting into his locker, he writes the combination in a different base for each number of the combination (base 2 for the first number, base 3 for the second number, base 4 for the third number). The three numbers for Hizny's combination are written 1100, 101010, and 101, but are not necessarily in order. If the highest possible number on the lock is 60, what is Hizny's base 10 locker combination?

A) $42 - 36 - 17$ B) $42 - 5 - 12$ C) $21 - 10 - 20$ D) $42 - 17 - 36$ E) NOTA

8. What is the partial fraction decomposition of the following: $\dfrac{4x^3 - 11x^2 - 33x - 12}{x^4 + 2x^3 - 5x^2 - 6x}$

A) $\dfrac{2}{x} + \dfrac{1}{x+1} + \dfrac{4}{x+3} - \dfrac{3}{x-2}$ B) $\dfrac{1}{x} + \dfrac{2}{x+1} - \dfrac{5}{x+2} + \dfrac{7}{x-1}$ C) $\dfrac{3}{x} + \dfrac{2}{x+1} - \dfrac{38}{5x+15} - \dfrac{7}{4x-8}$

D) $\dfrac{1}{x} + \dfrac{2}{x+1} + \dfrac{3}{x+3} - \dfrac{4}{x-2}$ E) NOTA

9. When adding two positive even integers, what are all possible outcomes for the sum?

A) all positive numbers B) all positive integers C) all positive even integers
D) all positive even integers also divisible by 4 E) NOTA

10. The sum of the first n terms of a sequence of integers is $n^2 + 1$. How many terms in the sequence are even?

A) 5 B) 12 C) 1 D) 13 E) NOTA

11. Find the determinant of the matrix A that satisfies $A\begin{bmatrix} 2 & 1 \\ 4 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$.

A) 1   B) -1 C) 0   D) 4   E) NOTA

12. A three-digit number has its digits reversed to form a new three-digit number. The product of the two three-digit numbers is 58843. Find the sum of the digits of either three-digit number.

A) 9   B) 15   C) 12   D) 10   E) NOTA

13. How many real numbers satisfy the equation $\Big| \big| \big| |x-1|-2\big|-3\big|-2\Big| = 1$?

A) 8   B) 16   C) 9   D) 15   E) NOTA

14. Find the sum: $\dfrac{12}{32} + \dfrac{8}{32} + \dfrac{5}{32} + \dfrac{3}{32} + \dfrac{1.75}{32} \ldots + \dfrac{n+2}{2^{n+2}} + \ldots$

A) $\dfrac{1}{2}$ B) 1   C) $\dfrac{3}{2}$ D) 2   E) NOTA

15. Emily likes to do problems, not in base 10, but in her favorite base. On her homework, she has written the equation $\sqrt{2424} = 42$. In base 10, the equation is not true, but in Emily's favorite base, the equation is true. What is Emily's favorite base?

A) 6   B) 9   C) 11   D) 13   E) NOTA

16. Austin, Garrett, Sascha, and Takaaki are standing at the vertices and co-vertices of the ellipse given by the equation $81x^2 + 100y^2 - 486x + 1000y + 3085 = 0$. They are wondering how much area there is which is enclosed by the ellipse but not enclosed by the rhombus with vertices where they are standing. Can you help them? How much area is that?

A) $\dfrac{8\pi - 16}{5}$ B) $8\pi - 16$ C) $\dfrac{16\pi - 32}{5}$ D) $4\pi - 8$ E) NOTA

17. A palindrome is an integer whose digits are read the same forward and backward. How many positive palindromes less than 1000 have square roots that are also palindromes?

A) 2   B) 4   C) 5   D) 6   E) NOTA

18. Jaewon rolls ten regular six-sided dice and notices that the product of the rolled faces is $2^5 3^5$. What is the largest possible sum of the faces Jaewon rolled?

A) 25   B) 30   C) 35   D) 40   E) NOTA

19. Find the values of x that satisfies $\cos\left(\dfrac{4\pi}{3} - \cos^{-1} x \right) = x$.

A) $\pm\dfrac{1}{2}$ B) $-\dfrac{\sqrt{3}}{2}$ C) $\pm\dfrac{\sqrt{3}}{2}$ D) $-\dfrac{1}{2}$ E) NOTA

20. Let A, B, and C be nonzero rational numbers. If the function $f(x) = Ax^2 + Bx - \dfrac{16}{27}$ has A and B as its roots, then what is the numerical value of B ?

A) $-\dfrac{4}{3}$ B) $-\dfrac{32}{27}$ C) $-\dfrac{8}{9}$ D) unable to be determined   E) NOTA

21. Find the value of x such that $\displaystyle\sum_{i=2}^{x} \frac{1}{\log_i 2} = \frac{1}{\log_{5040} 2}$.

A) 5040   B) 100   C) 7   D) 101   E) NOTA

22. The area enclosed by the graphs of $y = 2 - 3|x+1|$ and $y = k$ is equal to 7. Find the value of k .

A) $2 + \sqrt{21}$ B) $2 - \sqrt{21}$ C) $2 \pm \sqrt{21}$ D) $1 - \sqrt{21}$ E) NOTA

23. Find the sum of the solutions to the equation $\sin 3x \cos 3x = \frac{1}{4}$ on the interval $[0, \pi)$.

A) $\dfrac{3\pi}{2}$ B) $\dfrac{5\pi}{2}$ C) 2π D) 3π E) NOTA

24. Let $\lfloor x \rfloor$ represent the largest integer n such that $n \le x$. There is no solution to the equation $x\lfloor x \rfloor = 2008$. In fact, what is the last year c before 2008 in which there was a solution to the equation $x\lfloor x \rfloor = c$?

A) 1989   B) 1979   C) 1984   D) 1980   E) NOTA

25. What is the probability that when dealt two cards each from a standard deck of 52 cards, Lucy and Nikki have the same rank on each others' cards? (for example, both have a 10 and a 2 or both have two jacks)

A) $\dfrac{116}{20825}$ B) $\dfrac{73}{4165}$ C) $\dfrac{29}{4165}$ D) $\dfrac{146}{20825}$ E) NOTA

Tiebreakers

TB1. The first term of a geometric sequence is 1 and the last term of the sequence is 16. If the sequence consists only of integers, how many such sequences exist?

TB2. If 99 bottles of Coke are on the wall, 99 bottles of Coke, and one is taken down and passed around, how many bottles of Coke remain on the wall? Write your answer in scientific notation.

TB3. A clock has a minute hand of length 4 inches and an hour hand of length 3 inches. At what times between 3 o'clock and 4 o'clock is the distance between the tips of the hands equal to $\sqrt{13}$ inches?