2008 Hoover HS Math Tournament Algebra II Ciphering

Practice: Find the sum of the positive integral factors of 2008. 3780

1.1 How many vertical asymptotes does the function $f(x)=\dfrac{x^2-x-2}{x^3-4x^2+x+6}$ have? 1

1.2 What 2×2 matrix, when multiplied on the left of any 2×2 matrix A, interchanges the rows of A? $\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$

1.3 Find the coefficient of the x^5 term in the expansion of $(2x+3)^9$. 326592

1.4 Solve the system $\begin{cases}x+y+2z=1\\ x+2y+z=2\\ 2x+y+z=3\end{cases}$. $\left(\dfrac{3}{2},\dfrac{1}{2},-\dfrac{1}{2}\right)$

1.5 How many pairs of numbers have a greatest common divisor of $5!$ and a least common multiple of $10!$? 8

2.1 A fair, six-sided die has faces numbered $1,2,2,5,7,$ and 9. If this die is rolled three times and the sum of the faces rolled is noted, how many different sums are there? 22

2.2 If a, b, and c are distinct numbers such that $a^3+3a+14=0$, $b^3+3b+14=0$, and $c^3+3c+14=0$, find $\dfrac{1}{a}+\dfrac{1}{b}+\dfrac{1}{c}$. $-\dfrac{3}{14}$

2.3 Find the length of the latus rectum of the ellipse with foci at the points $(2,2)$ and $(-2,2)$ and passing through the point $(3,1)$. $\dfrac{\sqrt{26}+5\sqrt{2}}{3}$

2.4 The cost of manufacturing Durka fudge is \$36 fixed plus \$7 per piece. The revenue generated is $\$(x+2)$ for each of x items sold. How many pieces of Durka fudge must be sold to break even (when cost and revenue are equal)? 9

2.5 Find the value of $\left(\left(\left(\left((2008)^{2007}\right)^{2006}\right)^{2005}\right)\ldots\right)^{0}+\left(\left(\left(\left((0)^{1}\right)^{2}\right)^{3}\right)\ldots\right)^{2008}$ 1

3.1 Find the sum: $\dfrac{2}{3}-\dfrac{5}{9}+\dfrac{8}{27}-\dfrac{11}{81}+\ldots$ $\dfrac{5}{16}$

3.2 Find the value of x that satisfies $\sqrt{x!(x+1)!}=x(x+1)$. 3

3.3 Find the area of the circle through the points $(10,-2)$, $(1,5)$, and $(6,4)$ 65π

3.4 What is the absolute value of the complex number that satisfies $z-3i=3z+\overline{z}$, where $\overline{z}$ represents the conjugate of z? 3

3.5 Find the sum of the squares of the roots of the equation $x^3-7x^2+25x-9=0$. -1

4.1 Find the values of x in the interval $[0,2\pi]$ such that $\cos x+1=\sin x$. $\dfrac{\pi}{2},\pi$

4.2 If the probability of being dealt at most three threes in five cards of a standard 52-card deck of cards is given as $\dfrac{a}{b}$, where a and b are relatively prime, find the value of $b-a$. 1

4.3 An increasing sequence of positive integers is defined in the following way: for any term beyond the third term, $a_{n+3} = a_n + a_{n+1} + a_{n+2}$. If $a_{10} = 392$, what is the value of a_3? 5

4.4 Let $\lfloor x \rfloor =$ the largest integer n such that $n \le x$. The x-values of the points of intersection of $y = \left| \frac{\lfloor 3x \rfloor}{2} \right|$ and $y = 1 - \left| \frac{\lfloor 6x \rfloor}{2} \right|$ are all real numbers in the interval $a \le x < 0$. Find the numerical value of a. $-\frac{1}{6}$

4.5 Find the smallest positive integer x such that $(2008!)(2007!)(2006!)(2005!)x$ is a perfect square. 251753

E.1 What must x equal if $1 + x + x^2 + x^3 + ... = 10$? $\frac{9}{10}$

E.2 Find the distance between the points $(e, \sqrt{e})$ and $(1, -\sqrt{e})$. $e+1$