## Geometry Test VHHS Math Tournament 2007

1. Anne buys a perfectly spherical scoop of ice cream with a radius of 3 cm. Instead of eating it, though, she watches it melt. How many centimeters tall would a cone with the same diameter as the scoop of ice cream have to be to perfectly contain the melted ice cream?

A. 9 B. 12 C. 18 D. 6 E.NOTA

2.  $\overrightarrow{AB}$  and  $\overrightarrow{BC}$  are both tangent to circle *O*. *E* lies on  $\overrightarrow{AB}$  and *D* lies on  $\overrightarrow{BC}$ .  $\overrightarrow{ED}$  is tangent to circle *O*. If  $\overrightarrow{AB} = 20$ , what is the perimeter of  $\triangle EBD$ ?

A. 55 B. 50 C. 45 D. 40 E. NOTA

3. Find the distance from point B to the centroid of triangle ABC if A (10,12), B (-3,8), and C (2,1).

A.  $\sqrt{37}$  B. 6 C  $\sqrt{39}$  D. 4 E. NOTA

4. What is the maximum rectangular area in square meters that can be enclosed by 240 meter of fencing if the fence were to be constructed next to a river so that no fence was needed on the river boundary?

A. 14400 B. 1600 $\sqrt{3}$  C. 6400 D. 7200 E. NOTA

5. Cubes of edge length 1 are stacked to form a cube of edge length 5. A square hole with side length 1 is made through the center of each face of the cube through the entire figure. The resulting figure is dipped in paint then disassembled into the original unit cubes. Find the number of unit cubes with exactly two sides painted.

A. 24 B. 60 C. 68 D. 84 E. NOTA

6. Find the equation of the line that is the perpendicular bisector of the segment with endpoints (-5,7) and (-2,1). Your answer should be in slope intercept form.

A. 
$$y = \frac{1}{2}x + \frac{15}{4}$$
 B.  $y = \frac{1}{2}x + \frac{9}{4}$  C.  $y = -2x - 3$  D.  $y = \frac{1}{2}x + \frac{23}{4}$  E. NOTA

7. Triangle ABC has sides BC = 6, AC = 9 and  $\angle ACB = 60^{\circ}$ , What is the area of this triangle?

A. 
$$\frac{27\sqrt{3}}{2}$$
 B.27 C.  $\frac{18\sqrt{95}}{5}$  D.  $\frac{25\sqrt{7+2\sqrt{3}}}{2}$  E. NOTA

8. An ant is standing at the top of a cylindrical can of Diet Pepsi Max and needs to reach the UPC code at the bottom of the can on the opposite side. How far in inches does the ant have to crawl if the circumference of the can is 6 inches and the height is 8 inches?

A. 10 B.  $\sqrt{73}$  C. 5 D.8 E. NOTA

9. Alex owns a zebra named Wayne. When Alex goes on vacation, he leaves Wayne at home and ties him to the corner of his house (pictured) with a 20 yard rope. What is the area (in square feet) of Wayne's feeding space while Alex is gone?



10. There is a cone with radius 9 inches and height 12 inches. A cut is made parallel to the base 4 inches from the tip of the cone. What is the volume in cubic inches of the frustum formed?

A.324  $\pi$  B. 252  $\pi$  C.216  $\pi$  D.312  $\pi$  E. NOTA

11. Jason and Matthew decide to have a race. They start side-by-side on the school track, which is a rectangular field with a semicircle on each of the long sides of the field. The rectangular field is 80 yards by 40 yards. If Matthew is twice as fast as Jason, how many yards will Jason have run when Matthew runs one more lap than Jason?

A.80+20  $\pi$  B.160+160  $\pi$  C. 160+40  $\pi$  D.80+80  $\pi$  E. NOTA

12. A cube of side length 3 in. has its corners cut off 1 in. from each vertex, so that each piece cut is a pyramid with a triangular base. What is the surface area in square inches of the resulting figure?

- A.  $24 + 16\sqrt{2} + 4\sqrt{3}$
- B.  $18 + 12\sqrt{2} + 4\sqrt{3}$
- $C.12 + 12\sqrt{2} + 3\sqrt{3}$
- D.  $24 + 16\sqrt{2} + 3\sqrt{3}$
- E. NOTA

13. Given a trapezoid has sides length of 1,2,3, and 4. Find the area of the trapezoid.

A. 4 B. 
$$8\sqrt{2}$$
 C.  $\frac{5\sqrt{2}}{3}$  D.  $\frac{10\sqrt{2}}{3}$  E. NOTA

14. A circle centered at O as shown below, with points A,M,C, and B on it. M is the midpoint on arc AC and  $MN \perp AO$ . What is the relationship between MN and AC?



A. 
$$MN = \frac{3}{5}AC$$
 B.  $MN = \frac{\sqrt{2}}{2}AC$  C.  $MN = \frac{1}{2}AC$  D.  $MN = \frac{\sqrt{3}}{3}AC$  E. NOTA

15. Let A be the number of circles that can be tangent to each of two intersecting lines. Let B be the number of common tangents that two externally tangent circles can have. The value A+B is between which two numbers, inclusive?

A. 1 and 4 B.5 and 10 C.11 and 15 D.16 and 20 E. NOTA

16. Find the volume of the figure determined by rotating the triangle with points (3,1); (3,5); and (6,1) around the *y*-axis?

A.12  $\pi$  B. 48  $\pi$  C. 84  $\pi$  D. 96  $\pi$  E. NOTA

17. Find the diameter of a circle whose segment of a 60° sector has a measure of  $\frac{32\pi - 48\sqrt{3}}{3}$ ?

A.  $\frac{64\pi - 96\sqrt{3}}{\pi}$  B.8 C.  $\frac{32\pi - 48\sqrt{3}}{\pi}$  D. 16 E. NOTA

18. Find the area bounded by the lines:  $\begin{cases} 4x - y = 7 \\ -x + 2y = 0 \end{cases}$ , and the *x*-axis.

A. 1 B.  $\frac{7}{8}$  C.  $\frac{7}{16}$  D.  $\frac{7}{24}$  E. NOTA

19. If there is a trapezoid with bases length 5, and 13 and side edges length  $2\sqrt{21}$  and 10, what is the height of this trapezoid?

A.  $2\sqrt{3}$  B.  $2\sqrt{6}$  C.  $5\sqrt{3}$  D.  $5\sqrt{6}$  E. NOTA

20. Three concentric circles of radii 2, 4, and 6 make up a dartboard. The point values assigned are 10, 20, 30 in order from outside to inside. Given that all throws hit the dartboard, what is the probability of hitting the area worth 10 points?

A. 
$$\frac{1}{3}$$
 B.  $\frac{1}{9}$  C.  $\frac{4}{9}$  D.  $\frac{5}{9}$  E. NOTA

21. Find the arc length of the following figure:  $2y = \sqrt{100 - 4x^2}$ A.  $5\pi$  B.10+5 $\pi$  C.  $10\pi$  D.20 $\pi$  E. NOTA 22. A sector is cut out from a circular sheet of paper, and the two edges of the cut are joined together (with no overlap) to form a cone. The cone formed has radius 6 and volume  $96 \pi$ . What is the number of degrees in the sector that was cut off?

A. 60 B.120 C.144 D.216 E. NOTA

23. Three circles each with radius 1 intersect each other in a way such that each circle goes through the centers of the other two circles. Find the area of this "three-circle" figure?

A. 
$$\pi + \sqrt{3}$$
 B.2  $\pi + 2\sqrt{3}$  C. 2  $\pi - \frac{\sqrt{3}}{2}$  D.3  $\pi - \sqrt{3}$  E. NOTA

24. A spider travels across its web (pictured). If it starts at point A and only travels up and to the right, in how many ways can it get to the fly at point B?



25. ABCD is a trapezoid. EF is its median. Find the length of segment GH. (Figure is not drawn to scale).





TB1: In  $\triangle$  ABC, the average of the measures of  $\angle$  A and  $\angle$  B is 55° and the average of the measures of  $\angle$  C and  $\angle$  A is 65°. Find the positive difference between the measures of  $\angle$  B and  $\angle$  C.

TB2: Find the maximum area of the rectangle with perimeter 25.

TB3: The sum of the diagonals of rhombus ABCD is 28 and m  $\angle$  CBA = 2 m  $\angle$  DAB. Find the perimeter of the rhombus.