Algebra II Test VHHS Math Tournament 2007

- 1. Find the number of terminating zeroes at the end of 2007!.
 - A) 497 B) 499 C) 500 D) 501 E) NOTA

2. What is the 3rd term of the expansion of
$$\left(\frac{x}{2}+3\right)^5$$
?
A) $\frac{45x^3}{8}$ B) $\frac{3x^2}{4}$ C) $\frac{9x^3}{8}$ D) $\frac{45x^3}{4}$ E) NOTA

3. What is the sum of the coefficients of $(9x-8y)^7$?

A) -1 B) 0 C) 1 D) 2 E) NOTA

4. In mad scientist Dr. Dedo's genetics lab, there are certain numbers of 7-legged frogs and 4-legged monkeys. Dr. Dedo counted the total number of legs (not including Dr. Dedo's) in the lab and found it to be 67. Given that there are an odd number of genetically enhanced animals, how many mutated monkeys are in his lab?

A) 5 B) 6 C) 7 D) 8 E) NOTA

5. A polynomial has only three roots: 3, -4, and -3. If P(0) = -72, find P(5).

- A) 144 B) 212 C) 288 D) 612 E) NOTA
- 6. In how many ways can 5 distinguishable beads be placed on a leather bracelet (there is no fixed point on the bracelet)?
 - A) 12 B) 24 C) 60 D) 120 E) NOTA

7. Liangsha decides to row upstream against the current ¼ of a mile before rowing back downstream to her camp, which is ¾ of a mile from her starting position. Later she discovers that it took the same amount of time for her upstream journey and her downstream journey. If she rows a constant rate throughout, how fast did Liangsha row if the total journey takes 18 minutes?

- A) 3 mph B) $\frac{25}{16}$ mph C) $\frac{25}{9}$ mph D) $\frac{17}{9}$ mph E) NOTA
- 8. If $\begin{cases} a^3 + b^3 = 468\\ (a+b)^3 = 1728 \end{cases}$, find *ab*, where *a* and *b* are both real numbers.
 - A) 35 B) 40 C) 45 D) 50 E) NOTA
- 9. What does $x^2 y^2 + 4x + 2y + 3 = 0$ graph?
 - A) ellipse B) hyperbola C) parabola D) circle E) NOTA

10. I have a ball. I drop it off a 60 feet tall building. The ball rebounds to 5/8 of its original height after hitting the ground. Find the total distance traveled by the ball before it comes to a rest (in feet).

- A) 260 B) 160 C) 320 D) 240 E) NOTA
- 11. Evaluate: $\sum_{n=1}^{8} 2n^2 + n + 1$. A) 444 B) 445 C) 450 D) 452 E) NOTA

12. If you are dealt 5 cards from a standard 52 card deck, what is the probability that you get a full house, 3 of a kind and 2 of a kind (Example QQQ88)?

A)
$$\frac{3}{5625}$$
 B) $\frac{6}{4165}$ C) $\frac{6}{4465}$ D) $\frac{7}{3485}$ E) NOTA

13. If $z_1 = 5 - 12i$ and $z_2 = 3 + 4i$, find $\left| \frac{z_1}{z_2} \right|$. A) $\frac{12}{7}$ B) 2 C) $\frac{12}{5}$ D) 3 E) NOTA

14. Mr. Taylor and Mrs. Tipton are playing a dice game. Mrs. Tipton is using a fair 6-sided die numbered from 1-6, but Mr. Taylor is using a loaded 6-sided die numbered from 1-6, such that the probability of rolling a 1 is $\frac{1}{36}$, rolling a 2 is $\frac{3}{36}$, rolling a 3 is $\frac{5}{36}$, and so on. If they roll at the same time, what is the sum of the expected values of the numbers shown on the dice?

A) 6 B)
$$\frac{251}{36}$$
 C) 7 D) $\frac{287}{36}$ E) NOTA

15. If f(ax) = af(x) for all real a, and f(3) = 2, what is f(13)?

A) $\frac{39}{2}$ B) $\frac{26}{3}$ C) $\frac{37}{6}$ D) $\frac{29}{3}$ E) NOTA

16. How many positive odd divisors does 6552 have?

A) 6 B) 12 C) 24 D) 36 E) NOTA

17. Let $M = (\log_2 3)(\log_3 4)(\log_4 5)...(\log_{2006} 2007)$. What is $\log_{2007} 8^M$?

18. Evaluate:
$$\sqrt{(\log_2 32768)(i+1)^8 + \sqrt{(\log_2 32768)(i+1)^8 + \sqrt{(\log_2 32768)(i+1)^8 + ...}}}$$

A) -8 *i* B) -15 C) 16 D) 24 E) NOTA

19. Solve for x over the field of complex numbers: $x^4 + 5x^2 + 9 = 0$

A)
$$\frac{\pm 1 \pm i\sqrt{11}}{2}$$
 B) $\frac{\pm 1 \pm i\sqrt{13}}{2}$ C) $\frac{\pm 1 \pm 4i}{2}$ D) $\frac{\pm 1 \pm i\sqrt{17}}{2}$ E) NOTA

20. Given:
$$_{A} = \begin{pmatrix} a & 2 & 5 \\ 0 & 1 & 4 \\ 2 & 3 & 6 \end{pmatrix} B = \begin{pmatrix} 1 & 5 & 3 \\ 2 & b & 7 \\ 1 & 0 & 5 \end{pmatrix} C = \begin{pmatrix} 10 & 7 & 43 \\ 6 & 1 & 27 \\ 14 & 58 & c \end{pmatrix}$$
. If $AB = C$, find $a + b + c$.
A) 57 B) 59 C) 67 D) 69 E) NOTA

21. Evaluate: $\sum_{n=2}^{16807} \log_7 \left(\frac{n-1}{n} \right)$ A) -5 B) -4 C) 4 D) 5 E) NOTA 22. Find the sum of the ordered triple (x, y, z) that solves $\begin{cases} 5x + 3y - 4z = 16\\ 2x - 4y + 3z = 11\\ x + 3y - 4z = 27 \end{cases}$

23.
$$\frac{1}{2} + \frac{1}{4} + \frac{1}{4} + \frac{3}{16} + \frac{5}{32} + \frac{1}{8} \dots = ?$$

A) 4 B) 3 C) e D) 2 E) NOTA

24. Set S contains elements with the following properties:

- I. 2 is in S
- II. If x is in S, then x+7 and 3x are in S
- III. No numbers other than those that satisfy conditions I and II are in S

What is the smallest integer greater than 2007 that is NOT in S?

A) 2011 B) 2010 C) 2009 D) 2008 E) NOTA

25. The asymptotes of $y = \frac{x^4 + 3x^3 - 15x^2 - 19x + 30}{x^3 - x^2 - 30x + 72}$ are in the form y = a, y = b, y = cx + d. Find the value of a + b + c + d.

A) -1 B) 3 C) 7 D) 11 E) NOTA

TB1. Given a dodecagon with side length 1 inscribed in a circle, find r^2 , where r is the radius of the circle.

TB2. Find the amplitude of the graph $y = [22\sin(2x)][17\cos(2x)]$

TB3. How many sides does a chiliagon have?