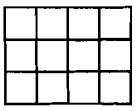
2007 Hoover High School Mathematics Tournament
Comprehensive Written Test

1. A square matrix A is idempotent if $A^2 = A$. Which of the following is not an idempotent matrix?

A) $\begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix}$ B) $\begin{bmatrix} 0 & 3 \\ 0 & 1 \end{bmatrix}$ C) $\begin{bmatrix} -3 & 6 \\ -2 & 4 \end{bmatrix}$ D) $\begin{bmatrix} 2 & 2 \\ -3 & 4 \end{bmatrix}$ E) NOTA

2. A 4x3 grid is made up of twelve squares, each with area 1. How many shapes that have an area of 3 may be constructed from squares in the grid if each shape must consist of squares in which each square shares at least one edge with at least one other square in the shape?



A) 10 B) 26 C) 34 D) 220 E) NOTA

3. Tyler H. started reading a 696-page book on page X and finished reading the book several hours later. He noticed that it took 1851 digits to number all of the pages he read. What is the value of X?

A) 66 B) 68 C) 70 D) 72 E) NOTA

4. Charlie draws a perfect circle on the ground. Katy then draws a perfect equilateral triangle with side length 24 inscribed in Charlie's circle. Russell then draws a perfect circle inscribed in Katy's triangle. What is the positive difference between Charlie's circle's radius and Russell's circle's radius?

A) $4\sqrt{3}$ B) $24-12\sqrt{2}$ C) $8\sqrt{3}$ D) $24-8\sqrt{3}$ E) NOTA

5. Hao multiplies a 2x2 matrix by a 2x6 matrix. He transposes that matrix, then removes the last two rows of the result. Hao now doubles the resulting matrix's dimensions and removes the first four rows of that result. What are the final dimensions of all of Hao's operations?

A) 6x6 B) 6x4 C) 4x6 D) 4x4 E) NOTA

6. Tyler C. is tethered to an upper corner of a building, 15 ft. x 20 ft. with a height of 12 ft., by a rope 15 ft. in length. He is going to jump off the roof onto the ground. What is the area of the region on the ground where he could possibly land?

A) $\frac{675\pi}{4}$ B) $\frac{243\pi}{4}$ C) 108π D) 81π E) NOTA

7. Eight actors are trying out for the part of Grandpa Seth in a traveling production of *Troll 2*, and five actresses are trying out for the part of Holly in the same production. In how many different ways can be two characters be cast?

A) 40 B) 13! C) ${}_{13}P_2$ D) $\binom{13}{2}$ E) NOTA

8. Find the length of the interval of the solution to the inequality $\frac{x-4}{7x+6} \geq 3$.

A) $\frac{6}{7}$ B) $\frac{34}{7}$ C) $\frac{13}{70}$ D) ∞ E) NOTA

9. Rectangle $ABCD$ has sides $\overline{AB}$ of length 6 and $\overline{BC}$ of length 8. Diagonal $\overline{AC}$ is drawn, and altitudes from points B and D are drawn to $\overline{AC}$ and intersect $\overline{AC}$ at points E and F, respectively. Find the length of $\overline{EF}$.

A) $\frac{14}{5}$ B) 3 C) $\frac{16}{5}$ D) $\frac{18}{5}$ E) NOTA

10. If a and b are nonnegative integers with $a < b$, find the largest value of $1+\frac{a!}{b!}$.

A) $\frac{4}{3}$ B) 2 C) $\frac{3}{2}$ D) $\frac{5}{4}$ E) NOTA

11. Find the sum of the imaginary solutions to the equation $x^3 - ix^2 - (3+i)x + (2+2i) = 0$.

A) $1-i$ B) i C) $-i$ D) $1+i$ E) NOTA

12. The area bounded by the x-axis, the line $x = 11$, and the piecewise function $f(x) = \begin{cases} \frac{3}{8}x, & 0 \le x \le 8 \\ 2x-13, & 8 < x \le 11 \end{cases}$ is rotated about the x-axis. Find the volume of the resulting solid.

A) 141π B) 114π C) 297π D) 216π E) NOTA

13. Find the remainder when 223^{2007} is divided by 19.

A) 7 B) 13 C) 18 D) 3 E) NOTA

14. Define the n th hyperfactorial $H(n)$ to be $1^1 2^2 3^3 \ldots n^n$. How many hyperfactorials have four digits?

A) 3 B) 0 C) 2 D) 1 E) NOTA

15. Swaroop stands at the point $(-4,6)$, Jamie stands at the point $(0,10)$, and Ryan stands at the point $(3,2)$. Tausif, wanting to be the center of attention, wants to stand at the centroid of the triangle made from his three friends. At what point should Tausif stand?

A) $\left(\frac{1}{3},6\right)$ B) $\left(-\frac{1}{2},6\right)$ C) $\left(-\frac{1}{2},9\right)$ D) $(0,5)$ E) NOTA

16. Find the value of $\sqrt{8+\frac{8}{\sqrt{8+\frac{8}{\sqrt{8+\frac{8}{\ldots}}}}}}$

A) $\sqrt{5}-1$ B) -2 C) $\sqrt{5}+1$ D) 2 E) NOTA

17. Nathan found a polynomial that has $\cos 10^\circ$ as a zero. However, he had to use the identity $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$ to verify that it was a zero. What was the polynomial that Nathan found?

A) $f(x) = 8x^3 - 6x - 1$ B) $f(x) = 8x^3 - 6x - \sqrt{3}$ C) $f(x) = 4x^3 - 6x$ D) $f(x) = 8x^3 - 6x - \sqrt{2}$ E) NOTA

18. Kevin and Danny are playing a game with a standard deck of 52 playing cards. Danny draws a card, and he doubles his money (gets his bet back plus an additional amount equal to his bet from Kevin) if he draws a jack or a diamond, but not if he draws the jack of diamonds. Kevin wins Danny's bet if Danny draws the jack of diamonds or any non-jack non-diamond. What is Kevin's expected value of the game if Danny bets \$52 on the game?

A) +\$40 B) +\$22 C) +\$44 D) +\$20 E) NOTA

19. Nisarg notices an interesting pattern concerning Pythagorean triples. If you have a Pythagorean triple with consecutive leg lengths, the hypotenuse may end in which of the following digits?

A) 1 B) 2 C) 3 D) 4 E) no Pythagorean triple with consecutive leg lengths ends in 1, 2, 3, or 4

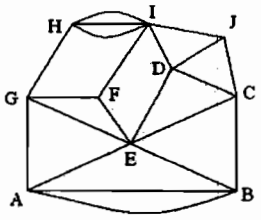
20. Find the smallest positive integer value of N that satisfies $\sum_{i=1}^{N} (\log_{10!} i) > 3$

A) 12 B) 16 C) $(10!)^3 + 1$ D) 21 E) NOTA

21. Sarah and Éva are standing on the surface of a cube with vertices $(0,0,0),(0,4,0),(4,0,0),(4,4,0),(0,0,4),(0,4,4),(4,0,4)$, and $(4,4,4)$ (assume gravity propels them toward the cube). If Sarah is standing at the point $(2,3,0)$ and Éva is standing at the point $(3,3,4)$, how far must Sarah walk to meet Éva?

A) 7 B) $\sqrt{17}$ C) $\sqrt{37}$ D) $3\sqrt{3}$ E) NOTA

22. Buzz is trying to get home, which is located at point J on the diagram. In the spirit of taking up too much time, he would like to cross each path in the diagram exactly once. From which point in the diagram, other than J, must Buzz start?



A) H B) F C) A D) G
E) NOTA

23. If a stick of butter has adjacent edges defined by vectors $\vec{u}=\langle 1,0,5\rangle, \vec{v}=\langle 2,-1,4\rangle$, and $\vec{w}=\langle -3,6,0\rangle$, in standard position, find $\vec{u}\cdot(\vec{v}\times\vec{w})$

A) 20 B) 24 C) 16 D) 18 E) NOTA

24) Find the sum: $\sum_{n=1}^{2007}\left\lceil \sqrt{2n+0.25}-0.5 \right\rceil$, where $\lceil x \rceil$ is the least integer greater than or equal to x.

A) 85344 B) 89313 C) 84777 D) 88746 E) NOTA

25. Which of the following is a logical conclusion from the following statements?: 1) If Zachary is cool, then Steven is not cool; 2) If Zachary is not cool, then I'm a monkey's uncle; 3) Steven is cool

A) Zachary is cool B) Steven is not cool C) I'm not a monkey's uncle D) I'm a monkey's uncle E) NOTA

TB1. Find the largest solution to $\sin x = 0.5, x \le \frac{241\pi}{12}$

TB2. "Finished files are the result of years of scientific study combined with the experience of years." Find the number of f's in this problem.

TB3. Is the statement in the outermost double quotes true, is it false, or does it not have a truth value?
"The statement `The statement "This statement is false." is false.' is false."