2007 Hoover HS Math Tournament Comprehensive Ciphering

Practice: Find the smallest integer in the following list: $2007, \sqrt{2007}, \sqrt[3]{2007}, \ldots, \sqrt[2007]{2007}$ 2007

1.1 When n is any positive integer, which of the following are always even? I. $n^{n+1}+(n+1)^n$; II. $(n+1)!$; III. 2^n+7n II

1.2 Find the infinite sum: $\sum_{n=2}^{\infty}\frac{1}{n!}$ $e-2$

1.3 What nonzero value of c gives the function $f(x)=2cx^2-cx+4$ only one real root? 32

1.4 A regular icosahedron has an edge of length $\sqrt[4]{12}$. Find its surface area. 30

1.5 Kus tells Buzz to add 2 to the denominator of $\frac{1}{x}$. Buzz, not listening, instead adds 2 to the fraction $\frac{1}{x}$. Conveniently, the result is the same. What is the numerical value of the result? 1

2.1 Find the tangent of the acute angle formed by the lines $y=x$ and $y=3x$ $\frac{1}{2}$

2.2 Two balls are sitting on a level table and tangent to each other. A third ball, also sitting on the table, is nestled between them so that it is tangent to both of the other balls. If the lengths of the two smaller circles' radii are 2 and 3, find the length of the radius of the largest circle. $30+12\sqrt{6}$

2.3 If ΔABC is an equilateral triangle with side length 4 and D & E are the midpoints of sides AB and AC, respectively, find the sum of the perimeter and the area of ΔDEB. $4+3\sqrt{3}$

2.4 Find the value of $\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{\cdots}}}}}$ $-1+\sqrt{3}$

2.5 Find the reflection of the point $(-1,2)$ through the line $7x+2y=10$. $\left(\frac{129}{53},\frac{158}{53}\right)$

3.1 If $\left(\tan\left(\sin^{-1}\frac{8}{x}\right)\right)^2=\frac{A}{B}$, where A and B are relatively prime positive integers, find the smallest positive integral value of x that satisfies $\sqrt{A+B}=x$. 9

3.2 If $x+\frac{1}{x}=3$, find $\frac{(x^3-\sqrt{2}x^{1.5}+1)(x^3+\sqrt{2}x^{1.5}+1)}{x^3}$ 18

3.3 Find the cosine of the largest angle of a triangle with side lengths 7, 8, and 11. $-\frac{1}{14}$

3.4 Find the equation of the line through the point $(1,2)$ that cuts off the least area in the first quadrant. $y=-2x+4$

3.5 Find the smallest positive integer that leaves a remainder of 4 when divided by either 6 or 13, and a remainder of 6 when divided by 17. 550

4.1 In ΔABC, $\angle A=45^\circ$, $|AB|=10\sqrt{2}$, and $|BC|=2\sqrt{26}$. Find all possible lengths of the altitude of the triangle to side AB. $4\sqrt{2}$ or $6\sqrt{2}$

4.2 If θ is the angle of inclination of the conic section defined by $3x^2-4xy+5y^2-y+2=0$, where $0<\theta<\frac{\pi}{2}$, find $\tan\theta$. $\frac{-1+\sqrt{5}}{2}$

4.3 Find the determinant: $\begin{vmatrix} 0 & 2 & 1 & 3 \\ 4 & 1 & -2 & 1 \\ 5 & -1 & -3 & 2 \\ 0 & 4 & 7 & 4 \end{vmatrix}$ 175

4.4 Write in $a+bi$ form: $\left(\frac{\sqrt{6}-\sqrt{2}}{4}+\frac{\sqrt{6}+\sqrt{2}}{4}i\right)^{2007}$ $\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i$

4.5 Solve for x: $1_2+3_4+5_6+\ldots+(2n-1)_{2n}+\ldots+99_{100}=x_{10}$. 15850

E.1 Factor completely over integers: x^4+1024 $\left(x^2+8x+32\right)\left(x^2-8x+32\right)$

E.2 Find the sum of the positive integral factors of 2007. 2912