2007 Hoover High School Mathematics Tournament
Algebra 2 Written Test

1. Find the value(s) of x that satisfy the equation $\log x + \log(x-2) = \log(x^2 - 2x)$.
A) $x > 2$ B) $0 < x < 2$ C) $x < 0$ D) all real numbers E) NOTA

2. Solve for x: $\dfrac{136x^3 - 1088}{8x - 16} = 0$
A) 2 B) $2, -1 \pm \sqrt{3}i$ C) $-1 \pm \sqrt{3}i$ D) no solutions E) NOTA

3. For every one point Elizabeth scores in Algebra 2 trashcan basketball, Asif then scores three points. If Asif gives Elizabeth 60 free points, how many additional points will Elizabeth score when their scores are first tied?
A) 10 B) 20 C) 30 D) 40 E) NOTA

4. Find the constant term in the expansion of $\left(x^8 - \dfrac{2}{x^6}\right)^7$
A) -35 B) 35 C) -560 D) 560 E) NOTA

5. Simplify: $(\csc\theta - \cot\theta)(\csc\theta + \cot\theta)$
A) 1 B) $\tan^2\theta$ C) $1 + \sin\theta$ D) $\dfrac{1+\cos\theta}{2}$ E) NOTA

6. Ankita's water heater needs repair. The repairman says it will cost \$300 to fix the unit, which currently costs \$75 per year to operate. Ankita could buy a new energy-saving water heater for \$525, including installation, and the new water heater would save 60% on annual operating costs. After how many years would the new water heater pay for itself?
A) 5 years B) 4.5 years C) 4 years D) 5.5 years E) NOTA

7. Which system has no solution?
A) $\begin{cases} x + 3y = 9 \\ 3x + y = 10 \end{cases}$ B) $\begin{cases} 3x = 12 - y \\ y - 6x = 8 \end{cases}$ C) $\begin{cases} x - 7 = -3y \\ 3y = x + 9 \end{cases}$ D) $\begin{cases} 9y = 3x + 3 \\ 3y - 2x = 9 \end{cases}$ E) NOTA

8. A license plate consists of three letters followed by three numbers, all of which are different. Find the probability that when you apply for a license plate, you receive one with only odd digits.
A) $\dfrac{1}{8}$ B) $\dfrac{1}{18}$ C) $\dfrac{1}{16}$ D) $\dfrac{1}{12}$ E) NOTA

9. If the probability that Leandra likes white yams more than orange yams is 0.8, and 0.6 is the probability that if Leandra likes any white thing better than its orange counterpart then she will fall asleep in class, what is the probability that Leandra fall asleep in class?
A) 0.6 B) 0.8 C) 0.48 D) 0.2 E) NOTA

10. Don't worry, Leandra will definitely fall asleep in class; however, the last question was purely hypothetical. What is the product of the real values of x in the equation $x^5 + 7x^4 + 11x^3 + 13x^2 + 28x - 60 = 0$?
A) 60 B) -60 C) 12 D) 15 E) NOTA

11. Find the sum of the following series, assuming it exists: $\dfrac{z^2}{y\sqrt{x}} + \dfrac{z^6}{y^3x^{1.5}} + \dfrac{z^{10}}{y^5x^{2.5}} + \ldots + \dfrac{z^{4n-2}}{y^{2n-1}x^{n-0.5}} + \ldots$
A) $\dfrac{xyz^2}{y^2\sqrt{x} - z}$ B) $\dfrac{yz^2\sqrt{x}}{xy^2 - z^4}$ C) $\dfrac{yz^4\sqrt{x}}{xy^2}$ D) $\dfrac{xyz^4}{xy^2 - z^2}$ E) NOTA

12. Matt wants to know how he can get the most interest added onto his savings account. He deposits \$2007 and has options of A) 5% interest compounded quarterly; B) 6% interest compounded annually; C) 7% interest compounded semiannually; or D) 6% interest compounded quarterly. In which option should Matt invest to maximize his interest?

A) A B) B C) C D) D E) two of them are equal

13. Stephen has a really,-really,-superball (RRSB) which rebounds to $\frac{5}{3}$ of its original height. If Stephen drops the RRSB from a height of 4 feet, after how many bounces will the RRSB first be higher than the roof of Hoover High School, which has a height of 55 feet?

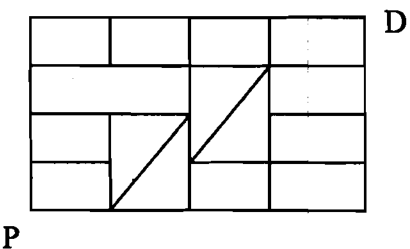
A) 5 B) 6 C) 7 D) 8 E) NOTA

14. Find the value of $\left(\frac{1+i}{1-i}\right)^{2007}$.

A) $1+i$ B) $1-i$ C) i D) $-i$ E) NOTA

15. Mack is standing at point P in the diagram to the right, and he wants to reach Salman, who is standing at point D. In how many ways can Mack reach Salman if he may only move up, to the right, or both simultaneously?

A) 26 B) 31 C) 35 D) 27 E) NOTA



16. If a solution to $x^6 = -64i$ may be written as $a + ai$, where a is a positive real number, then $a = ?$

A) $\sqrt{6} \pm \sqrt{2}$ B) $\sqrt{2}$ C) $\sqrt{3}-1$ D) $\pm\sqrt{2}$ E) NOTA

17. Let $X =$ the geometric mean of 2 and 12.5 and $Y =$ the arithmetic mean of 9 and 13. Find the harmonic mean of X and Y.

A) 55 B) $\frac{55}{2}$ C) $\frac{55}{4}$ D) $\frac{55}{8}$ E) NOTA

18. Randall and Helen leave school at the same time and drive in opposite directions. Randall drives 15 mph faster than Helen. They drive for 30 minutes, and their combined distance traveled is 45 miles. How fast is Helen driving?

A) 60 mph B) 37.5 mph C) 45 mph D) 52.5 mph E) NOTA

19. When a certain two-digit number's digits are reversed, then the resulting number is equal to the original number minus five times the sum of the digits of the original number. Find the original number.

A) 62 B) 71 C) 80 D) there is no such number E) NOTA

20. The number 2007! has prime factorization written in the form $2^{a_1}3^{a_2}5^{a_3}..b^{a_n}$, where the prime factors are written in increasing order and b is the last prime factor in the prime factorization. Find $b + a_5 + a_{13}$.

A) 2251 B) 2551 C) 2665 D) 2668 E) NOTA

21. In the equation $\sqrt{a+4\sqrt{5}} + \sqrt{a-4\sqrt{5}} = \sqrt{b+4\sqrt{5}}$, where a and b are positive integers, b must be equal to what number?

A) 10 B) 20 C) 5 D) 4 E) NOTA

22. If Joyce decomposed $\frac{x^2+29}{(x+1)^2(x^2+4)}$ into partial fractions, what did she get for the numerator of the term with (x^2+4) as its denominator?

A) 2 B) 6 C) $-2x-3$ D) $2x+6$ E) NOTA

23. Using the fact that $\sin 10^\circ \sin 50^\circ \sin 70^\circ = \frac{1}{8}$, find the value of $\prod_{k=1}^{18} \sin(20k-10)^\circ$

A) $-\frac{1}{2^{16}}$ B) $\frac{1}{2^{16}}$ C) $-\frac{1}{2^{18}}$ D) 0 E) NOTA

24. If $(-4,-5)$ is the focus with smaller y-value of the conic section given by the equation $\frac{(y-1)^2}{Z}-\frac{(x+4)^2}{16}=1$, where $Z>0$, find the vertex with the larger y-value.

A) $(-4,5)$ B) $(-4,7)$ C) $\left(-4,1+2\sqrt{5}\right)$ D) $\left(-4,-7+2\sqrt{5}\right)$ E) NOTA

25. Alex noticed that $X=\frac{\sum_{i=2007}^{10,000} i!}{2007!}$ is a positive integer. If x_k is the remainder when X is divided by k, find $\sum_{a=2008}^{2013} x_a$.

A) 3 B) 2007 C) 2020 D) 3989 E) NOTA

Tiebreakers

TB1. Rudy & Tucker are wrestling; the probability that Rudy wins is $\frac{1}{e}$, and the probability that Tucker wins is $\frac{1}{\pi}$. Only one dog can win. What is the probability that neither wins?

TB2. The 15th, 16th, and 17th decimal place digits of π add up to the 18th digit. In addition, the 19th digit, 4, is half the 18th digit. The 15th digit equals the 17th digit, and any two of the 15th through 17th digits have a sum larger than the remaining third digit of the group. What is the three-digit number made up of the 15th through 17th digits of π?

TB3. You get this one free if you just write the word BLINDDARMENTZÜNDUNG, complete with umlaut and in all capitals and spelled correctly.