2007 Hoover HS Math Tournament Algebra II Ciphering

Practice: Find the smallest integer in the following list: $2007, \sqrt{2007}, \sqrt[3]{2007}, ..., \sqrt[2007]{2007}$ 2007

1.1 What is the remainder when 23^3 is divided by 7? 1

1.2 A triangle has side lengths of 2 and 5, and the interior angle formed by those sides is $60°$. Find the length of the third side. $\sqrt{19}$

1.3 How many positive integral divisors does $20^3 + 7^3$ have? 10

1.4 A set of 2007 numbers, $\{x_1, x_2, ..., x_{2007}\}$ has a mean value of $\mu = 2007$. Find the value of $\sum_{i=1}^{2007}(x_i - \mu)$ 0

1.5 If $\log x^2 + \log x^3 = 3$, then $x = ?$ $\sqrt[5]{1000}$

2.1 The number 9991 may be factored as $A * B$, where A and B are proper positive factors of 9991. If $A > B$, find B. 97

2.2 How many distinct permutations are there for the word SEPPUKU? 1260

2.3 If right triangle ABC has hypotenuse 11 and area 12, find the perimeter of ABC. 24

2.4 Find the value of $\sqrt{7 - \sqrt{7 - \sqrt{7 - \sqrt{7 - ...}}}}$ $\frac{-1+\sqrt{29}}{2}$

2.5 How many values of x satisfy the equation $\frac{\frac{2}{x+2}}{\frac{1}{x+2}+\frac{2}{x}} = \frac{2}{3}$? 0

3.1 Which letter has a larger numerical value: $A = \frac{-1+\sqrt{31}}{2}$ or $B = \cos 75° + \sin 90° - \cos 180°$? A

3.2 The range of the function $y = \frac{3x^2}{x^2 - 4}$ is $y \le a$ or $y > b$, where a and b are real numbers. Find $(b+a)^{b-a}$. 27

3.3 If $(x-2)$ is a factor of $f(x) = x^3 + mx^2 - 10$, find the value of m. $\frac{1}{2}$

3.4 The graph of $|y| + |Bx + 1| = 5$, where $B > 0$, encloses an area of 1 unit. Find the value of B. 50

3.5 If $\left(\tan\left(\sin^{-1}\frac{8}{13}\right)\right)^2 = \frac{A}{B}$, where A and B are relatively prime positive integers, find $\sqrt{A+B}$. 13

4.1 Find the sum of the squares of the roots of the function $f(x) = x^3 + 4x^2 - 3x + 5$. 22

4.2 Define the sets X and Y in the following way: $X = \{x \in \mathbb{R} \mid 7 < x^2 < 25\}$ and $Y = \{y \in \mathbb{R} \mid 4 < y^3 < 100\}$. How many functions are there with domain X and range a subset of Y? 81

4.3 Find the length of the latus rectum of the conic section given by the equation $3x^2 - 7y^2 + 12x - 14y - 16 = 0$

$\frac{6\sqrt{7}}{7}$

4.4 Find the entry in the third column, second row of A^{-1} for the matrix $A = \begin{bmatrix} 8 & 0 & 1 \\ 0 & 3 & 0 \\ 2 & 0 & -1 \end{bmatrix}$. 0

4.5 Find the coordinates of the focus with the larger y-value for the conic section given by the equation $36x^2 + 25y^2 - 216x + 50y - 551 = 0$ $\left(3, -1+\sqrt{11}\right)$

E.1 If A^T represents the transposed matrix of $A = \begin{bmatrix} 6 & -7 \\ -3 & 4 \end{bmatrix}$, and if $B = \left(A^T\right)^{-1}$, find $|B|$. $\frac{1}{3}$

E.2 Find the remainder when the third perfect number is divided by the product of the first and second perfect numbers. 160