2006 Hoover HS Math Tournament
Geometry Written Test

1. Suppose an isosceles triangle has a perimeter of 1. Let a be the length of one of the two congruent sides. Which represents all the possible values of a?

A. $\frac{1}{2} < a < 1$ B. $\frac{1}{4} < a < \frac{1}{2}$ C. $0 < a < \frac{1}{2}$ D. $\frac{1}{4} < a < 1$ E. NOTA

2. A, B, and C are points on a circle. Assume A and B are endpoints of a diameter, $AB = 3$, and $AC = 2$. Find BC.

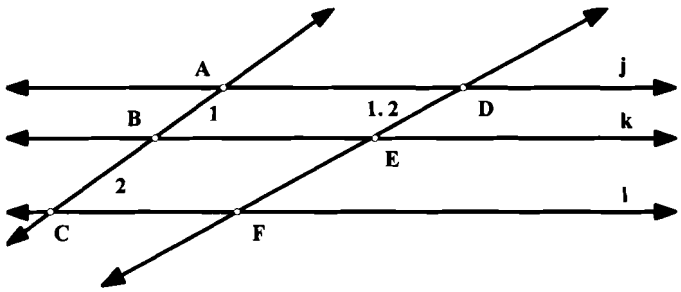
A. $\sqrt{5}$ B. $\sqrt{3}$ C. 1 D. $\frac{3}{2}$ E. NOTA

3. Let A be the set of whole numbers x with the property that there exists a circle and three lines (all coplanar) so that the intersection of the circle with the union of the lines consists of exactly x points. How many elements does A have?

A. 6 B. 5 C. 4 D. 7 E. NOTA

4. In the following figure lines j, k, and l are parallel, $AB = 1$, $BC = 2$, and $DE = 1.2$. Find DF.

A. 3.2 B. 4 C. 3.4 D. 3.6 E. NOTA



5. The volume of a sphere is increased by 33.1%. By what percent is the radius increased?

A. 8% B. 9.1% C. 10% D. 11% E. NOTA

6. A rectangle has a width of 1. A similar rectangle is removed from one end, and the remaining rectangle has area 5. Find the length of the original rectangle.

A. 5 B. $\frac{5+\sqrt{29}}{2}$ C. $\frac{4+\sqrt{34}}{2}$ D. 4 E. NOTA

7. A certain circle has a radius equal to an integer. On this circle there is a circular sector with area $\frac{25\pi}{12}$ and perimeter $\frac{60+5\pi}{6}$. Find the radius of the circle.

A. 10 B. 12 C. 6 D. 5 E. NOTA

8. A point in the first quadrant on the circle with the equation $x^2 + y^2 = 1$ has a tangent line that makes an angle of 30° with the x-axis. Find its coordinates.

A. $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ B. $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ C. $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ D. $\left(\frac{\sqrt{3}}{3}, \frac{\sqrt{6}}{3}\right)$ E. NOTA

9. Suppose that for a certain triangle, the incenter is located on one of the medians. Then the triangle must be:

A. Equilateral B. Scalene C. Isosceles D. Acute E. NOTA

10. A right circular cone is sliced by a plane perpendicular to its base. If the plane does not contain the vertex and the intersection of the cone, and the plane consists of more than one point, then the intersection is a portion of a:

A. parabola B. ellipse C. hyperbola D. circle E. NOTA

11. A regular tetrahedron has edges of length 6. A right prism of height 3 has equilateral triangles of side 6 as bases. Find the ratio of the volume of the tetrahedron to the volume of the prism.

A. $\frac{2}{3}$ B. $\frac{\sqrt{6}}{3}$ C. $\frac{3}{2}$ D. $\frac{2\sqrt{6}}{9}$ E. NOTA

12. A prism has a height of 3 and a base which is a regular octagon with sides of 2. Find its volume.

A. 24 B. $8+24\sqrt{2}$ C. $24+24\sqrt{2}$ D. $8\sqrt{2}$ E. NOTA

13. A farmer wants to fence in a pigpen along a stone wall which will serve as part of a boundary. The rest of the pen is circular with a radius of 2 yards and with the center of the circle 1 yard away from the wall and within the pen. Find the amount of fence required for the circular part of the boundary.

A. $\frac{8\pi}{3}$ yd. B. 2π yd. C. $\frac{10\pi}{3}$ yd. D. $\frac{4\pi}{3}$ yd. E. NOTA

14. Find the x-coordinate of the point on the positive x-axis that is a distance 1 from the line with equation $2y = x$.

A. $\sqrt{5}$ B. 2 C. $\sqrt{3}$ D. 3 E. NOTA

15. Find the distance from the point $(1,1,1)$ to the plane containing $(1,0,0)$, $(0,2,0)$, and $(0,0,4)$.

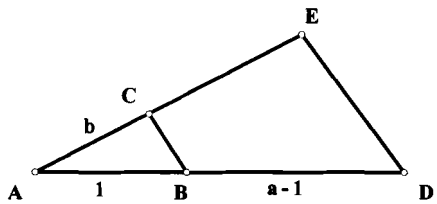
A. $\frac{1}{2}$ B. $\frac{\sqrt{21}}{7}$ C. $\frac{\sqrt{21}}{3}$ D. $\frac{\sqrt{7}}{3}$ E. NOTA

16. A solid object is sitting on the xy -plane so that its cross-sectional area at height z, sliced parallel to the xy -plane, is $\frac{\pi(6-z)^2}{4}$. Find its volume. (Hint: It has the same cross-sectional area as a certain cone.)

A. 12π B. 6π C. 36π D. 18π E. NOTA

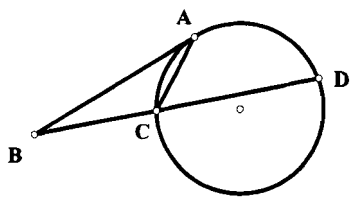
17. In the following figure assume the distances as labeled, and that $\overline{BC}$ is parallel to $\overline{DE}$. Which length is equal to ab ?

A. *CE* B. *AE* C. *ED* D. *BC* E. NOTA



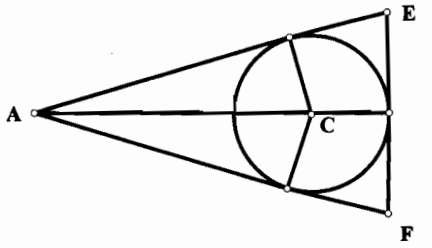
18. In the following figure, assume $\overline{BA}$ is tangent to the circle, $m\angle ABC = 12^\circ$, and $m\angle ADC = 25^\circ$. Find the measure of $m\angle CAD$.

A. 100° B. 90° C. 118° D. 112° E. NOTA



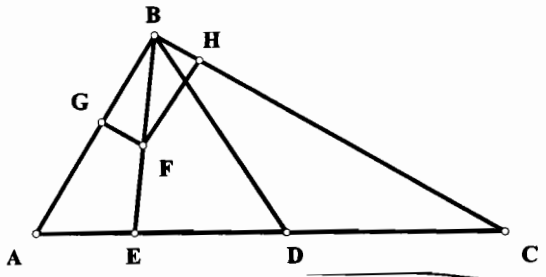
19. In the following figure, C is the center of the circle and segments $\overline{AE}$, $\overline{EF}$, and $\overline{FA}$ are tangent to the circle at the points shown. Suppose the radius of the circle is 4 and that $AC = 6$. Find EF.

A. $6\sqrt{3}$ B. 7 C. $5\sqrt{2}$ D. $8\sqrt{5}$ E. NOTA



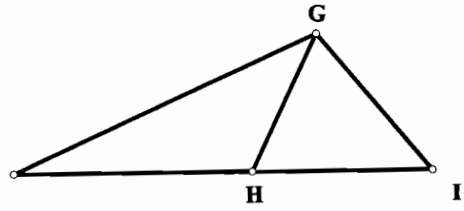
20. In the following figure assume: D is the midpoint of $\overline{AC}$, $m\angle ABE = m\angle CBD$, $AB = 3$, $BC = 4$, $\overline{FG}$ is perpendicular to $\overline{AB}$, $\overline{FH}$ is perpendicular to $\overline{CB}$, and $FH = \frac{3}{2}$. Find FG.

A. $\frac{9}{8}$ B. $\frac{3}{4}$ C. $\frac{4}{3}$ D. $\frac{5}{16}$ E. NOTA



21. In the following triangle $JG = 4$, $GI = 3$, $HI = 2$, $JI = \frac{14}{3}$, $m\angle GHI = 76^\circ$, and $m\angle HIG = 57^\circ$. Find $m\angle JGI$.

A. 94° B. 90° C. 86° D. 47° E. NOTA



22. The perimeter of a right triangle is 6, and its area equals one of its legs. Find the length of the shortest leg.

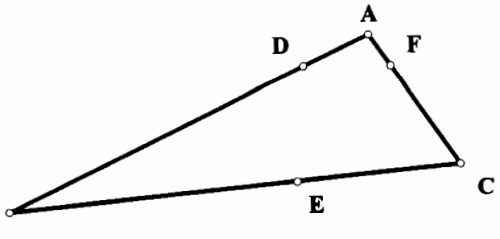
A. $\frac{3}{2}$ B. 2 C. $\frac{5}{2}$ D. 3 E. NOTA

23. The point P is located in the interior of a rectangle such that the distance from P to a corner is 5 and the distance to the opposite corner is 14. If the distance from P to a third corner is 10, find the distance to from P to the remaining corner.

A. 9 B. 11 C. 8 D. 12 E. NOTA

24. In the following figure, $BE = 3$, $EC = 2$, $CF = \frac{3}{2}$, and $FA = \frac{1}{2}$. The segments $\overline{AE}$, $\overline{BF}$, and $\overline{CD}$ are concurrent if and only if $\frac{AD}{AB}$ equals:

A. $\frac{3}{11}$ B. $\frac{2}{9}$ C. $\frac{2}{3}$ D. $\frac{2}{11}$ E. NOTA

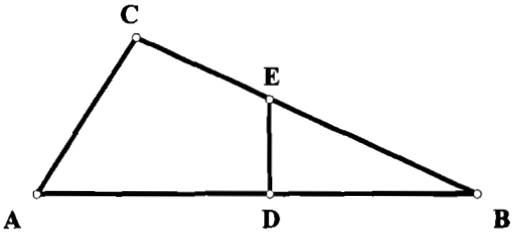


25. Suppose distinct lines l and m are parallel, and P is a point. Let Q be the reflection of P with respect to l, and then let R be the reflection of Q with respect to m. Suppose $PR = 10$. Find the distance between the two lines.

A. 5 B. 10 C. 15 D. 20 E. NOTA

TB 1. In the figure, $m\angle ACB = 90^\circ$, $AD = DB$, $m\angle BDE = 90^\circ$, $AB = 20$, and $AC = 10$. Find the area of the quadrilateral $ACED$.

A. 75 B. 58.5 C. 48 D. 37.5 E. NOTA



TB 2. Suppose two different spheres and a plane intersect in a finite number of points. The possibilities for the number of intersections are:

A. 0 and 2 only B. 0 and 1 only C. 1 and 2 only D. 0, 1, and 2 E. NOTA

TB 3. Find the area of the region above the line $y = 1$ and below the circle of radius 2 centered at the origin.

A. $\frac{4\pi}{3} - 1$ B. $\frac{4\pi}{3} - \sqrt{3}$ C. $\frac{2\pi}{3} - \frac{\sqrt{3}}{2}$ D. $\frac{\pi}{2} - \sqrt{3}$ E. NOTA