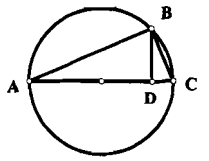
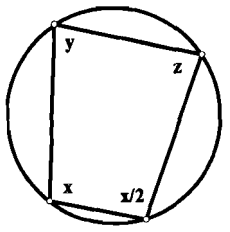
2006 Hoover HS Math Tournament
Geometry Ciphering

1.1 In the following figure $\overline{AC}$ is a diameter of the circle, $m\angle ADB = 90^\circ$, and $CD = 1$. If $BD = h$, which segment on the figure has length h^2. Ans: AD

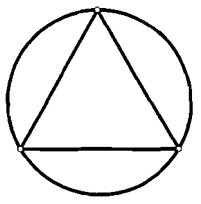


1.2 In the following, the variables represent angle measurements. Find $y - \frac{z}{2}$. Ans: 90°



1.3 A sphere of radius $r < 1$ is concentric with a sphere of radius 1, and the volume of the region between the spheres is $\frac{\pi}{3}$. Find r. Ans: $\frac{\sqrt[3]{6}}{2}$

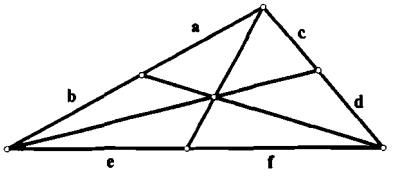
1.4 In the following figure there is an equilateral triangle inscribed in a circle. The sides of the triangle have length 6. The region between the triangle and the circle consists of 3 congruent regions. Find the area of one of these regions. Ans: $4\pi - 3\sqrt{3}$



1.5 Let A be the set of whole numbers " x " with the property that there exists two tangent circles and a line (all coplanar) so that the line intersects the union of the circles in exactly " x " points. How many elements are in set A ? Ans: 5

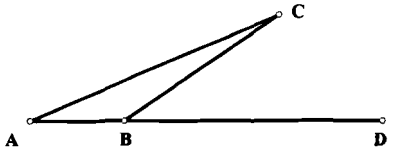
2.1 In the following figure the variables represent the lengths of the segments as indicated. Suppose $\frac{a}{b} = \frac{c}{d}$.

Find $\frac{e}{f}$. Ans: 1

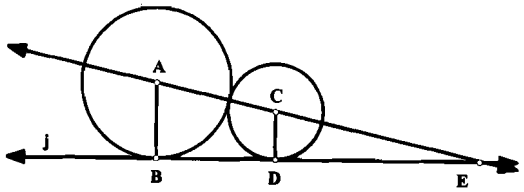


2.2 In the following figure, $AB = 1$, $BC = 5$, and $m\angle CBD = 30^\circ$. Find the area of $\triangle ABC$.

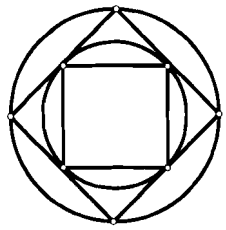
Ans: $\frac{5}{4}$



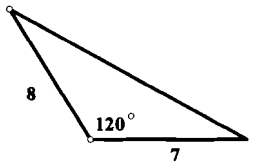
2.3 In the following figure the two circles are tangent, the line j is tangent to both circles, $AB = 4$, and $CD = 1$. Also the centers of the circles are A and C. Find AE. Ans: $\frac{20}{3}$



2.4 The following figure shows a square inscribed in a circle, which is inscribed in a square, which is (in turn) inscribed in a circle. If the area of the larger circle is 1, find the area of the smaller square. Ans: $\frac{1}{\pi}$



2.5 Find the area. Ans: $14\sqrt{3}$

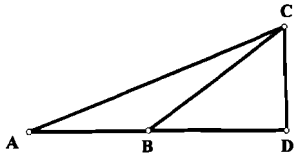


3.1 Suppose a triangle with sides 12, 16, and 20 is inscribed in a circle. Find the area of the circle. Ans: 100π

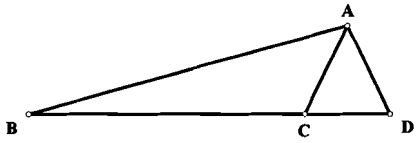
3.2 Suppose two circles of radius 1 are coplanar and intersect. Suppose further that the intersection of the two interiors has a boundary with a perimeter of $\frac{2\pi}{3}$. How far apart are the centers of the two circles? Ans: $\sqrt{3}$

3.3 Find $a > 0$ so that $(0,0,0)$ is a distance 1 from the plane passing through $(a,0,0)$, $(0,a,0)$, and $(0,0,2a)$. Ans: $\frac{3}{2}$

3.4 In the figure $AC = 5, AB = 2, m\angle ADC = 90^\circ$, and the area of $\triangle ABC$ is 3. Find BD. Ans: 2

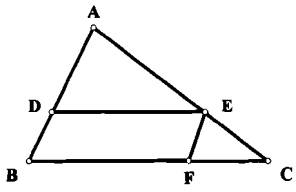


3.5 In the following figure, $\measuredangle ACB = 115^\circ, AC = AD$, and $\frac{AD}{CD} = \frac{AB}{CB}$. Find $m\angle BAC$. Ans: 50

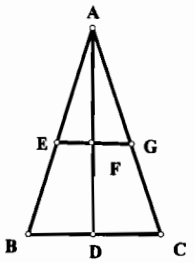


4.1 Find the distance from $(1,0)$ to the line with the equation $y = \sqrt{3}x$. Ans: $\frac{\sqrt{3}}{2}$

4.2 In the following figure, $\triangle ABC$ has area 42, $\overline{DE}$ is parallel to $\overline{BC}$, $\overline{EF}$ is parallel to $\overline{AB}$, and $AD = 2DB$. Find the area of $\triangle EFC$. Ans: $\frac{14}{3}$



4.3 In the following figure, $AB = AC$, $BD = DC$, $AD = 1$, and $\overline{EG}$ is parallel to $\overline{BC}$. Also, assume that the area of $\triangle AEG$ is one-half that of $\triangle ABC$. Find DF. **Ans:** $\frac{1}{2}\left(2-\sqrt{2}\right)$



4.4 The area of the circle of intersection of two spheres of radius 1 is $\frac{\pi}{4}$. Find the distance between the centers of the spheres. Ans: $\sqrt{3}$

4.5 The base of a prism consists of n contiguous isosceles triangles, each with a height of 1 and the two congruent sides $\frac{3}{2}$. If the height of the prism is n, and the volume of the prism is $8\sqrt{5}$, find n. Ans: 4