2006 Hoover High School Mathematics Tournament
Comprehensive Written Test

1. Find the value of x that satisfies the equation $x^{\ln 2} = 8$.

A) e^8 B) e^2 C) e^3 D) $(2\sqrt{2})^e$ E) NOTA

2. A group of people consists of 5 women and 4 men; among them are Rui Rick Wu, Eva, Nisarg, Jessi, and Melanie. How many committees of any number of people can be formed from this group of people given that there must be the same number of women and men in the committee? A committee may not consist of 0 members.

A) 120 B) 125 C) 126 D) 121 E) NOTA

3. Of the following answer choices, if Mingwei raised one of them to the second power, he would get the same thing as if he raised the same one to the seventh power. Which answer choice satisfies this relationship?

A) -1 B) cis 72° C) $\frac{1}{2}+\frac{\sqrt{3}}{2}i$ D) $\frac{\sqrt{3}}{2}-\frac{1}{2}i$ E) NOTA

4. If $a+b=7$ and $a^2+b^2=99$, find the value of $b^2\left(a^2+\frac{1}{b^2}\right)$.

A) 626 B) $\frac{25}{16}$ C) 625 D) 26 E) NOTA

5. Where defined, $2^{\log_{\cos x}(1-\sin x)+\log_{\cos x}(1+\sin x)} = ?$

A) $2^{\ln 3}$ B) 2 C) 4 D) $2^{\ln 2}$ E) NOTA

6. If Brandon expresses $.20\overline{06}$ as a fraction whose numerator and denominator are relatively prime, what is the sum of that numerator and denominator?

A) 2976 B) 5943 C) 5998 D) 119 E) NOTA

7. For $Z=\begin{bmatrix}4 & -1\\ 7 & 2\end{bmatrix}$, let $A=|Z|$, $B=|Z^{-1}|$, $C=|Z^T|$, and $D=$ the sum of the entries of Z. Evaluate $\left|\begin{bmatrix}A & B\\ C & D\end{bmatrix}^{-1}\right|$.

A) -45 B) $\frac{1}{179}$ C) 11 D) $\frac{1}{11}$ E) NOTA

8. Find the area enclosed by the ellipse whose equation is $(x-2)^2+2(y+1)^2=\sqrt{2}$.

A) $\pi\sqrt{2}$ B) $\frac{\pi}{\sqrt{2}}$ C) 2π D) π E) NOTA

9. Find the normal line to the equation $5x^2-3x+2y^2-4=0$ at the point $(1,1)$.

A) $y=\frac{4}{7}x+\frac{3}{7}$ B) $y=-\frac{7}{4}x+\frac{11}{4}$ C) $y=-\frac{4}{7}x+\frac{11}{7}$ D) $y=\frac{1}{7}x+\frac{6}{7}$ E) NOTA

10. Chords $\overline{AC}$ and $\overline{BD}$ meet at a right angle at point E in circle O. If $\overline{AE}$ has length 4, $\overline{CE}$ has length 15, and $\overline{BE}$ has length 6, find the radius of circle O.

A) 10 B) $\frac{\sqrt{446}}{2}$ C) $\frac{\sqrt{277}}{2}$ D) $\frac{\sqrt{377}}{2}$ E) NOTA

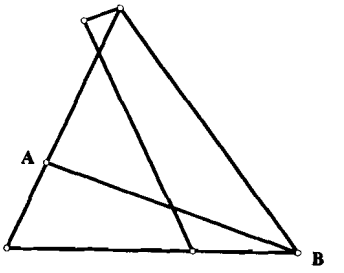
11. In how many ways can Wesley distribute five identical balls into three distinct boxes?

A) 21 B) 15 C) 10 D) 18 E) NOTA

12. A square of side length 2006 is inscribed in a circle. Another circle with radius 1003 has its center at one of the vertices of the square. Let $A =$ the area inside the larger circle, $B =$ the area inside the smaller circle, $C =$ the area inside the square, and $D =$ the area inside the square but outside the smaller circle. Find the ratio $\frac{D}{A+B+C}$.

A) $\frac{8-\pi}{4+5\pi}$ B) $\frac{16-\pi}{4+5\pi}$ C) $\frac{16-\pi}{16+12\pi}$ D) $\frac{8-\pi}{16+12\pi}$ E) NOTA

13 Buzz walks along the pathways shown to the right. When leaving any one of the eight vertices, he must walk in one direction until he reaches another vertex, at which point he may change directions. How many paths could Buzz take if he begins at A and wants to reach the deck of cards at point B, going through no vertex twice?



A) 13 B) 11 C) 12 D) 9 E) NOTA

14. Find the natural number A such that the greatest common divisor of A & 200 is 50, also the least common multiple of A & 200 is 1000.

A) 100 B) 50 C) 500 D) 250 E) NOTA

15. The function $f:\mathbb{R}\to\mathbb{R}$ is both an even and an odd function. Find $f(2006)$.

A) 2006 B) -2006 C) 0 D) 1 E) NOTA

16. Fifteen years ago, Shahein was fifteen years older than Hao. In two years, how many years older than Hao will Shahein be?

A) 32 B) 2 C) 15 D) 17 E) NOTA

17. Find the smallest real zero of the polynomial $g(x) = 2x^4 - 7x^3 - 11x^2 + 22x + 24$.

A) 4 B) 2 C) -1 D) 1 E) NOTA

18. How many distinct arrangements are there of the letters in UMMUMUMUUMM?

A) 252 B) 4620 C) 23100 D) 462 E) NOTA

19. Find the distance between the point $(7,2)$ and the line $x+\frac{3}{5}y=2$.

A) $\frac{31\sqrt{34}}{34}$ B) $\frac{31}{5}$ C) $\sqrt{29}$ D) $\frac{\sqrt{29}}{34}$ E) NOTA

20. Patrick and Sam are playing the game nines, and Patrick has a $4/7$ probability of winning a game. The first player to achieve three wins will win the best-of-five match. What is the probability that Sam wins two games but loses the match to Patrick?

A) $\frac{5760}{16807}$ B) $\frac{3456}{16807}$ C) $\frac{576}{16807}$ D) $\frac{2304}{16807}$ E) NOTA

21. Find the remainder when 2006^{2006} is divided by 23.

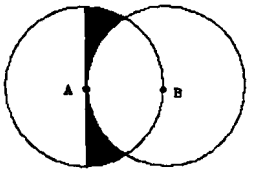
A) 4 B) 22 C) 1 D) 10 E) NOTA

22. A triangle has side lengths 7, 9, and 10. If the ratio of the area of the triangle's circumscribed circle to the area of its inscribed circle is written as a/b, where a and b are relatively prime, then find the sum of the digits of N, where $N = a - b$.

A) 15 B) 24 C) 18 D) 19 E) NOTA

23.

In the picture, A & B are centers of the circles. Each center lies on the other circle, and both circles have a radius of 1. Find the area of the shaded region.



A) $\frac{\pi}{6}$ B) $\frac{\sqrt{3}}{2}-\frac{\pi}{4}$ C) 1 D) $\frac{\sqrt{3}}{2}-\frac{\pi}{3}$ E) NOTA

24. n is the smallest positive integer such $n/2$ equals some integer squared, $n/3$ equals some integer cubed, $n/11$ equals some integer raised to the eleventh power, and $n/13$ equals some integer to the thirteenth power. The prime factorization of n is $p_1^{n_1} p_2^{n_2} \cdots p_k^{n_k}$, where $p_1, p_2, \ldots, p_k$ are distinct primes written in increasing order and $n_1, n_2, \ldots, n_k$ are distinct positive integers. Find the value of $\sum_{i=1}^{k}\left((p_i - n_i)(-1)^i\right)$.

A) 155 B) 153 C) 159 D) 158 E) NOTA

25. Find the sum: $\sum_{m=1}^{2006}\left(\sum_{n=1}^{2006}\left(\frac{2^n}{2^m + 2^n}\right)\right)$

A) 4,024,036 B) 1,006,009 C) 2,012,018 D) 1 E) NOTA

TB1. Write 2006_9 as a base 3 numeral.

TB2. On Planet Zulu, seven gnathemas equal three tontos, five tontos equal two crosks, and twelve crosks equal one foteshia. If Scott has twenty crosks, forty tontos, and one hundred forty gnathemas, and his favorite hoverboard costs four foteshias, how many foteshias does Scott have left after buying it?

TB3. If $\log_2 5 = 2.322$ and $\log_2 6 = 2.585$, find the value of $\log_4\left(\frac{36}{25}\right)$.