2006 Hoover HS Math Tournament Comprehensive Ciphering

Practice: Find the sum of all values of x for which $x+2=(x+1)^2$ -1

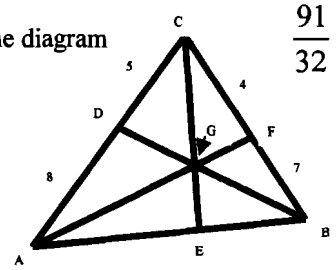
1.1 If z is a complex number such that $z^3 = 3-2i$, find $|z|$. $\sqrt[6]{13}$

1.2 At what time between 5 and 5:30 p.m. are the hour and minute hands of a clock separated precisely by a 35° angle? $5:20\frac{10}{11}$

1.3 Let $f(x)=[\![x]\!]$ be the greatest integer function. Find the infinite sum: $\sum_{n=1}^{\infty} f\left(\frac{1}{n}\right)$ 1

1.4 Find the area of a triangle with side lengths 1, e-1, and $\sqrt{e^2+2-2e}$ $\frac{e-1}{2}$

1.5 Find the ratio BG:GD in the diagram $\frac{91}{32}$



2.1 A 6-sided standard die is weighted so that the probability of rolling x is a/x, where a is a real number. If the only result of rolling the die is that is lands on one of the sides, what is the value of a? $\frac{20}{49}$

2.2 Let A=the number of distinct permutations of the word TINTINNABULATION, B=the product of positive primes less than 15, and C=the sum of the roots of the equation $3x^2-21x+1=0$. Find $\sqrt{\frac{AC}{B}}$. 1680

2.3 Find all solutions of the equation in the interval $[0,2\pi)$: $\sin(2t)+1 = \cos(t)+2\sin(t)$ $0, \frac{\pi}{6}, \frac{5\pi}{6}$

2.4 Let x=2006. Find $\frac{\binom{x+1}{x-1}}{\sum_{i=1}^{x} i}$ 1

2.5 Find the length of the latus rectum of the conic with equation $\frac{4}{7}x+1=-\frac{17}{13}y^2+67y$ $\frac{52}{119}$

3.1 Find the largest integer k such that 2006^k evenly divides 2006! 34

3.2 Find the value(s) of x such that $\sum_{i=1}^{\infty}\left(\frac{x}{2}\right)^i = \sum_{k=1}^{\infty}\left(\frac{1}{x}\right)^k$ $\pm\sqrt{2}$

3.3 Find the coefficient of the $x^2y^3z^5$ term in the expansion of $(2x+3y-z)^{10}$. -272160

3.4 How many of the eleven complex eleventh-roots of $-16i$ lie in the second quadrant of the complex plane? 2

3.5 Two dimensions of a rectangular prism are 4 and 6, and its space diagonal is 10. Find the volume of the prism. $96\sqrt{3}$

4.1 Solve for x:: $1+4x+9x^2+16x^3+\ldots = 9/2$ $\frac{1}{3}$

4.2 The first three terms of an arithmetic sequence are a, b, and c. The first three terms of a harmonic sequence are also a, b, and c. Find $\frac{ab}{c^2}$. 1

4.3 Solve for y: $(y+5)(y-9)=(y+7)(y-1)$ $-\frac{19}{5}$

4.4 Find the product, and write it as a base 2 numeral: $110100101_2 \times 1101_2$ 1010101100001_2

4.5 A sequence is defined recursively so that $a_1 = 4$ and $a_{n+1} = \begin{cases} 3a_n - 1, & n \text{ is even} \\ 3a_n + 1, & n \text{ is odd} \end{cases}$

Find the minimum value of $|2006 - a_i|$ where i is a positive integer. 973

E.1 Solve the inequality, and write your answer in interval notation: $\frac{x^3 - 1}{x - 1} \geq 0$ $(-\infty, 1) \cup (1, \infty)$

E.2 Find the largest integer value of the function $f(x) = -2x^2 - 3x + 3$ 4