2006 Hoover High School Mathematics Tournament
Algebra 2 Written Test

1. Find the quadratic function whose maximum value, 17, occurs when $x=-1$ and also goes through the point $(7,1)$.

A) $y=\frac{1}{4}\left(-x^2-2x+16\right)$ B) $y=-\frac{1}{4}(x+1)^2+17$ C) $y=-\frac{1}{4}(x-1)^2+17$ D) $y=\frac{1}{4}\left(-x^2-x+17\right)$
E) NOTA

2. Solve for x: $(\log 500-\log 5)(2x\log_3 27)=3\left(\frac{\log 46656}{\log 3+\log 2}\right)$

A) 2.5 B) 3 C) 2 D) 1.5 E) NOTA

3. How many vertical asymptotes does $f(x)=\frac{x^2-6x-16}{x^3-5x^2-2x+24}$ have?

A) 0 B) 1 C)2 D)3 E) NOTA

4. Solve for x: $(x-5)(x+2)=30$

A) 5, -2 B)10, 4 C) 8, -5 D) 8 E) NOTA

5. Which of the following is the conjugate of $\frac{3-2i}{12+5i}$?

A) $\frac{1}{169}\cdot\frac{3+2i}{12-5i}$ B) $\frac{1}{2-3i}$ C) $\frac{1}{169}\cdot\frac{3+2i}{12+5i}$ D) $\frac{1}{13}\cdot\frac{3+2i}{12-5i}$ E) NOTA

6. When $.\overline{2006}$ is expressed as a fraction whose numerator and denominator are relatively prime, what is the sum of that numerator and denominator?

A) 11994 B) 1981 C) 5998 D) 1999 E) NOTA

7. For $Z=\begin{bmatrix}4 & -1\\ 7 & 2\end{bmatrix}$, let $A=|Z|$, $B=\left|Z^{-1}\right|$, $C=\left|Z^T\right|$, and $D=$ the sum of the entries of Z. Evaluate $\left|\begin{bmatrix}A & B\\ C & D\end{bmatrix}^{-1}\right|$.

A) 1/179 B)-45 C) 11 D) $\frac{1}{11}$ E) NOTA

8. A right triangle has legs length 100_3 and 220_4. Find the perimeter of the triangle, written as a base 5 numeral.

A) 330_5 B) 240_5 C) 1210_5 D) 130_5 E) NOTA

9. Solve for x: $2^{4x}-2^{2x+\log_2 5}+4=0$

A) $\frac{1}{2},-\frac{1}{2}$ B) $\frac{1}{2}$ C) $-\frac{1}{2}$ D) no solution E) NOTA

10. If a, b, and c are the roots of the equation $f(x)=x^3+2x^2+5x-9$, find a+bc+c+ac+abc+b+ab.

A) -9 B) 12 C) -6 D) 6 E) NOTA

11.

How many routes are there for David, who is at point A, to reach Swaroop, who is at point B, if David may only move up or to the right?

A) 126 B) 464 C) 71 D) 97
E) NOTA

12. If $f(x) = 2006x^2 - 2006x + 2006!$, find the value of $\frac{f(x+h)-f(x)}{h}$, where $h \neq 0$.

A) $4012x + 2006h^2 - 2006$ B) $4012x + 2006h - 2006$ C) $4012x + 2006h - 2006!$

D) $\frac{2006h^2 - 2006h + 2006!}{h}$ E) NOTA

13. Zachary has a superball which rebounds to $7/8$ of its previous height. On the third bounce, the ball lands on a piece of gum and only rebounds to $1/8$ of its previous height. Find the total distance the superball has traveled, in feet, if it starts at a height of 4 feet and is caught at its highest point after the third bounce.

A) $\frac{1145}{128}$ B) $\frac{1729}{64}$ C) 60 D) $\frac{2241}{128}$ E) NOTA

14. Charlie has a bag containing 5 **red, 6 blue, and 4 white marbles**, and Russell has a standard 52-card deck. What is the probability of Katie drawing **a red marble out of the bag** if Jamie has already drawn a heart from the deck?

A) $\frac{1}{3}$ B) $\frac{1}{15}$ C) $\frac{1}{12}$ D) $\frac{5}{156}$ E) NOTA

15. Nathan has a \$1 bill, a \$2 bill, a \$5 bill, a \$10 bill, a \$20 bill, a \$50 bill, and a \$100 bill. He wants to give Stephanie and Tyler all of his money so that the difference of the amounts he gives them differs by \$25 or less. In how many different ways can Nathan give Stephanie and Tyler his money?

A) 2 B) 14 C) 12 D) 8 E) NOTA

16. Vinuta and Katy are standing on the points $(3,6)$ and $(3,-10)$, respectively. They decide that they want to meet at some point on the ellipse $\frac{(y+2)^2}{289} + \frac{(x-3)^2}{225} = 1$, so they both walk in a straight line toward that point. What is the sum of the distances that Vinuta and Katy must walk before they are reunited?

A) 34 B) 16 C) 30 D) 24 E) NOTA

17. Find the number of distinct permutations of the letters in the word STEVENHE, given that an E must come first.

A) 840 B) 6720 C) 4200 D) 2520 E) NOTA

18. Find the value of $\sqrt{2-\sqrt{2-\sqrt{2-...}}}$

A) 4 B) $\sqrt{3}$ C) 1 D) $\sqrt{2}$ E) NOTA

19. Christa, Danielle, and Jennifer are standing in the courtyard. The distance from Christa to Danielle is 7 meters, and the distance from Christa to Jennifer is 10 meters. If the angle between the line formed by Christa and Danielle and the line formed by Christa and Jennifer is 60°, find the distance, in meters, between Danielle and Jennifer.

A) 8 B) $\sqrt{42}$ C) $\sqrt{79}$ D) 7 E) NOTA

20. Of the following answer choices, If you raised one of them to the second power, you would get the same thing as if you raised the same one to the fifth power. Which answer choice satisfies this relationship?

A) -1 B) cis 72° C) $\frac{1}{2} + \frac{\sqrt{3}}{2}i$ D) $\frac{\sqrt{3}}{2} - \frac{1}{2}i$ E) NOTA

21. Find the value of x such that $1 + 2x + 3x^2 + 4x^3 + ... = 9$.

A) $\frac{2}{3}$ B) $\frac{4}{3}$ C) $\frac{1}{6}$ D) $\frac{2}{5}$ E) NOTA

22. Find the length of the latus rectum of the conic section with equation $\frac{(x+1)^2}{\sqrt{2}} + 2(y+1)^2 = \sqrt{2}$

A) $\sqrt{2}$ B) 2 C) $\frac{3}{2}$ D) $\frac{-1+\sqrt{5}}{2}$ E) NOTA

23. Tausif gives out awards with negative connotations. If you beg him for an award, he is half as likely to give you one. If you initially have a 75% chance of winning an award, after how many times of asking Tausif for an award does your probability of winning an award drop below 1%, virtually giving you no shot of winning such a prestigious award?

A) 8 B) 7 C) 3 D) 5 E) NOTA

24. Find the sum $\sum_{n=1}^{\infty} \frac{4}{n(n+4)}$

A) $\frac{5}{2}$ B) $\frac{8}{3}$ C) $\frac{25}{12}$ D) $\frac{13}{6}$ E) NOTA

25. Find the remainder when 0!+1!+2!+3!+4!+5!+.....+2006! is divided by 60.

A) 28 B) 42 C) 16 D) 34 E) NOTA

Tiebreakers

TB1. Find the shortest distance between the conic sections with equations $x^2+4x+y^2-6y+9=0$ and $x^2-6x+y^2+4y+4=0$.

TB2. If the probability of event A is 0.6, the probability of event B is 0.4, and the probability of A union B is 0.7, find the probability of A given that B has already occurred.

TB3. If $\log_2 5 = 2.322$, then log 2 can be written as 1/x. Find the value of x.