**Hoover High School Math Tournament**
**Geometry Exam**

**1.** The diagonals of a rhombus have lengths of 18 and 24. Find the area of a circle inscribed in this rhombus.

A. $54\pi$ B. $\frac{169}{9}\pi$ C. $\frac{1296}{25}\pi$ D. $\frac{289}{16}\pi$ E. NOTA

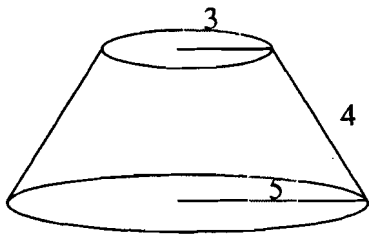
**2.** Find the shortest distance from the origin to the line segment containing $(2,0)$ and $(0,3)$.

A. $\frac{4\sqrt{13}}{13}$ B. $\frac{6\sqrt{13}}{13}$ C. $\frac{5}{3}$ D. $\frac{\sqrt{13}}{3}$ E. NOTA

**3.** Consider three different spheres in space. Let $A$ be the intersection of all three spheres. Which of the following are possibilities for the number of points in $A$? I. 3 II. 0 III. 2

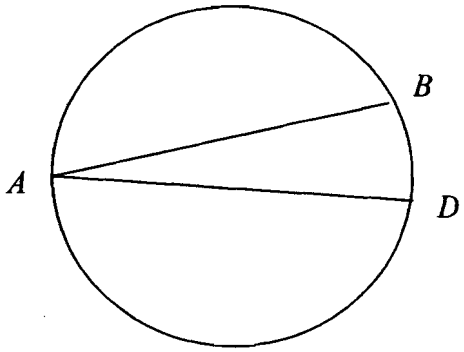
A. I and II B. I, II, and III C. I and III D. II and III E. NOTA

**4.** Determine the volume of the given figure.




A. $\frac{98\sqrt{3}}{3}\pi$ B. $75\pi$ C. $32\pi\sqrt{3}$ D. $\frac{119\sqrt{3}}{6}\pi$ E. NOTA

**5.** The following circle has radius 5, and $\overset{\frown}{BD}$, that is intercepted by $\angle BAD$, has length $\frac{5\pi}{3}$. Find $m\angle BAD$.




A. 25° B. 15° C. 30° D. 35° E. NOTA

**6.** A certain building is in the shape of a hemisphere of radius 20ft., with the circular base of radius 20ft. being the floor. A 24ft. board is leaning against the building so that the base of the board is on the ground. Assume the center of the base of the board is located 5ft. from the edge of the building. Find the distance from the center of the base of the board to the point where the board is touching the building.

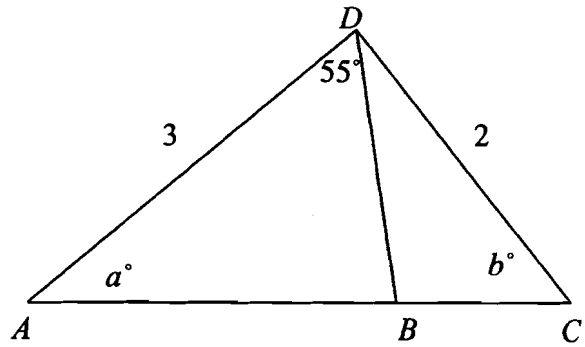
A. $9\sqrt{2}$ ft. B. 15 ft. C. 12 ft. D. $10\sqrt{2}$ ft. E. NOTA

7. Find the volume of a regular triangular pyramid if all edges are 3 centimeters long.

A. $2\sqrt{3}\ cm^3$ B. $\frac{3\sqrt{2}}{2}\ cm^3$ C. $\frac{9\sqrt{2}}{4} cm^3$ D. $\frac{2\sqrt{2}}{3}\ cm^3$ E. NOTA

8. In the following figure assume $a+b=70$ and $AB=\frac{3}{2}$.

Find $BC$.




A. $\frac{4}{3}$ B. $\frac{3}{2}$ C. $\frac{2}{3}$ D. 1 E. NOTA

9. A grapefruit in the shape of a sphere has a diameter of 4in. with a peel that is 0.25in. thick. Once the grapefruit is peeled, it has 12 congruent wedges. Find the volume of each wedge.

A. $\frac{511\pi}{576} in^3$ B. $\frac{9\pi}{10} in^3$ C. $\frac{29\pi}{36} in^3$ D. $\frac{343\pi}{576} in^3$ E. NOTA

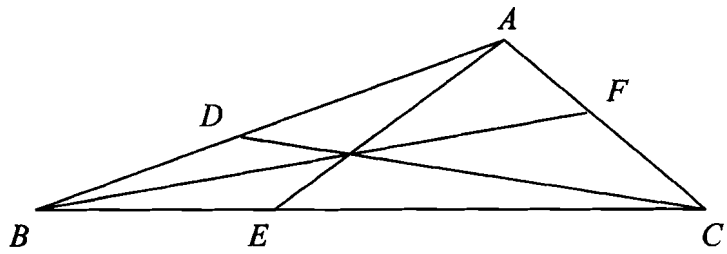
10. Find the distance from the origin to the plane through $(1,0,0)$, $(0,2,0)$, and $(0,0,3)$.

A. $\frac{7}{6}$ B. $\frac{6}{7}$ C. 2 D. $\frac{\sqrt{14}}{14}$ E. NOTA

11. The sum of the volumes of two spheres is $105\pi$. Their areas have the ratio of $4:9$. Find the volume of the smaller sphere.

A. $12\pi$ B. $20\pi$ C. $24\pi$ D. $32\pi$ E. NOTA

12. In the following figure $AB=3$, $BC=4$, $AC=2$, $BD=1$, and $CE=2$.
Find $CF$.

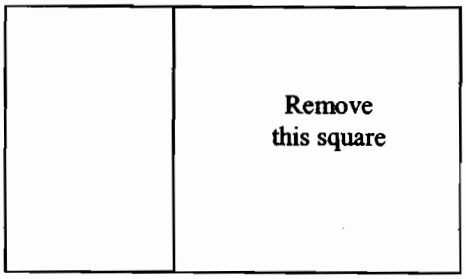



A. $\frac{4}{5}$ B. $\frac{3}{2}$ C. $\frac{3}{4}$ D. $\frac{2}{3}$ E. NOTA

**13.** A rhombus has interior angles of $30°, 150°, 30°$, and $150°$, and an area of $3$. Find its perimeter.

A. 9 B. $4\sqrt{6}$ C. $8\sqrt{3}$ D. $4\sqrt{3}$ E. NOTA

**14.** A certain rectangle has an area of $2\left(\sqrt{5}-1\right)$ and has the property that when a square is removed from one end, the remaining rectangle is similar to the original rectangle; see diagram. Find the length of the original rectangle.




A. $\sqrt{2}$ B. 2 C. $\sqrt{3}$ D. $\sqrt{5}+2$ E. NOTA

**15.** Find the volume of the solid formed by rotating the triangle with vertices $(-4,1), (-2,4)$, and $(-1,1)$ about the line $y=-3x-2$.

A. $\dfrac{27\pi\sqrt{10}}{10}$ B. $18\pi\sqrt{5}$ C. $\dfrac{18\pi\sqrt{3}}{5}$ D. $\dfrac{14\pi\sqrt{3}}{3}$ E. NOTA

**16.** Suppose a triangle has its incenter and its centroid located on one of its altitudes. Then the triangle:

A. must be equilateral B. must be scalene C. must be acute D. must be isosceles E. NOTA

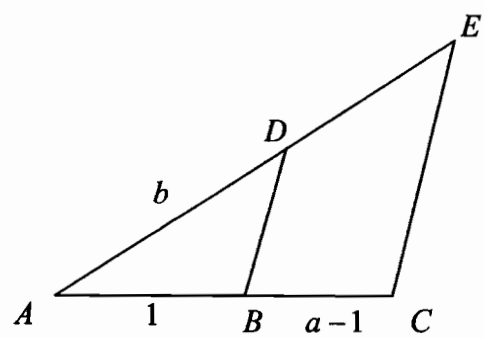
**17.** If the radius of a sphere is increased by 50%, by what percent is its volume increased?

A. 50% B. 150% C. 237.5% D. 337.5% E. NOTA

**18.** Suppose a circle is centered at the origin with radius larger than one, and the arc length of the arc of the circle above the line $y=1$, is $\dfrac{4\pi}{3}$. Find the radius of the circle.

A. $\sqrt{2}$ B. $\sqrt{3}$ C. 3 D. 2 E. NOTA

**19.** In the following figure assume $\overline{BD}$ is parallel to $\overline{CE}$. Find $AE$.

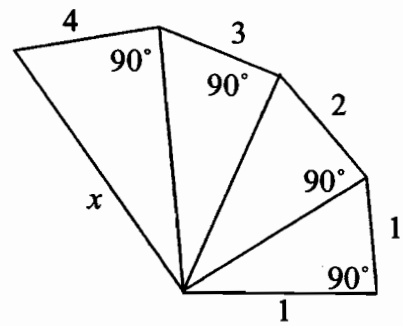



A. $ab$ B. $b(a-1)$ C. $a(b-1)$ D. $(a-1)(b-1)$ E. NOTA

**20.** Find the area of the circle $x^2 + y^2 - 4x + 6y - 3 = 0$.

A. $4\pi$ B. $8\pi$ C. $9\pi$ D. $12\pi$ E. NOTA

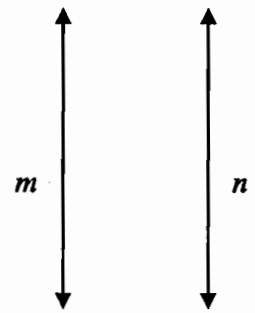
**21.** Consider the following figure.
Find x.




A. 2 B. $\sqrt{31}$ C. $4\sqrt{2}$ D. $2\sqrt{3}$ E. NOTA

**22.** Refer to the following figure. Assume lines $m$ and $n$ are both vertical and are separated by a distance of 2.

The reflection of a point $P$ with respect to $m$ followed by a reflection with respect to $n$ is the same as:




A. a translation to the right by 2 B. a translation to the left by 2 C. a translation to the right by 4 D. a translation to the left by 4 E. NOTA

**23.** Find the point on the circle centered at the origin of radius one that is closest to the point $(2,4)$.

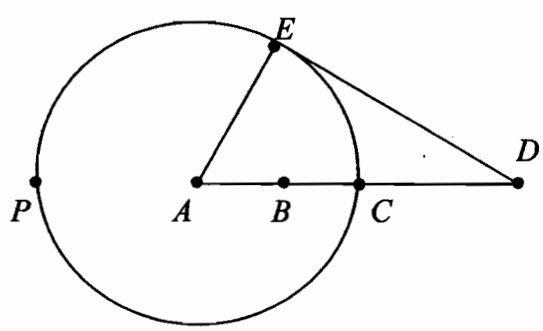
A. $\left(\frac{\sqrt{5}}{5}, \frac{2\sqrt{5}}{5}\right)$ B. $\left(\frac{\sqrt{5}}{10}, \frac{\sqrt{5}}{5}\right)$ C. $\left(\frac{2\sqrt{5}}{5}, \frac{3\sqrt{5}}{10}\right)$ D. $\left(\frac{\sqrt{15}}{5}, \frac{\sqrt{10}}{5}\right)$ E. NOTA

**24.** A prism has a height of $3\sqrt{3}$ and a regular hexagonal base with sides of length $2$. Find its volume.

A. $18\sqrt{3}$ B. 54 C. 81 D. $9\sqrt{6}$ E. NOTA

**25.** In the following figure, $A$ is the center of a circle of radius one, $AB = BC$, and $\overleftrightarrow{DE}$ is tangent to the circle at $E$.

Points $P, A,$ and $B$ are collinear, and $\frac{EC}{PE} = \frac{\sqrt{3}}{3}$. Find $AD$.




A. $2\sqrt{2}$ B. $\frac{\sqrt{3}}{2}$ C. $\sqrt{3}$ D. 2 E. NOTA

**TB-1.** Suppose $A$ and $B$ are two distinct points in plane $m$. Then the locus of points $P$ in $m$ so that $\frac{PA}{PB} = 2$ is a:

**TB-2.** From a given point, two tangent segments to a circle with a radius of 6 intercept a $120^\circ$ arc. Find the perimeter of the region bounded by the two tangents and the minor arc.

**TB-3.** Two tangent spheres with radii 6 and 10 rest on a table. How far apart are the points at which they touch the table?