Hoover High School Mathematics Tournament - March 5, 2005
Algebra 2 Written Test

(N.O.T.A. = none of the above)

1. Which of the following is true?

   (A) $\log(A + B) = \log A + \log B$ (B) $\frac{\log A}{\log B} = \log A - \log B$
   (C) $p \log A = \log(A^p)$ (D) $\log(AB) = (\log A)(\log B)$ (E) N.O.T.A.

2. Find x if $f(x) = \sqrt{x-2}$ and $f(x) = 2$.

   (A) 0 (B) 6 (C) undefined (D) 2 (E) N.O.T.A.

3. For $f(x) = x^2 + 15$ and $g(x) = \sqrt{3x}$, find $g(f(x))$.

   (A) $9x^2 + 5$ (B) $\sqrt{3x} + 2$ (C) $3x + 15$ (D) $\sqrt{3x^2 + 45}$
   (E) N.O.T.A.

4. Suppose f and g are real-valued functions with $f(x) = x^2$ and $g(x) = \sqrt{x}$. Find $f(g(-4))$.

   (A) 16 (B) -4 (C) undefined (D) 4 (E) N.O.T.A.

5. Find the slope of the line passing through the points $(1, 1)$ and (e, e^2).

   (A) $e + 1$ (B) $\frac{1}{e}$ (C) 3.71 (D) $e - \frac{1}{e}$ (E) N.O.T.A.

6. Which of the following equations does not describe the graph of a conic section or degenerate conic section?

   (A) $x = 0$ (B) $x + y = 0$ (C) $x^2 + y^2 = 0$ (D) $y = x^{-1}$
   (E) $y = x^{-2}$

7. How many integer solutions does the equation $\lfloor \frac{x}{7} \rfloor = 20$ have? ($\lfloor x \rfloor$ represents the largest integer $\leq x$.)

   (A) 4 (B) 8 (C) 3 (D) 6 (E) N.O.T.A.

8. If $h(x) = e^{x-3}$, find $h^{-1}(x)$.

(A) $\ln(3 - x)$ (B) $3 + \ln x$ (C) e^{3-x} (D) $\ln(3 - x)$
(E) N.O.T.A.

9. The product of an odd function $h(x)$ and an odd function $g(x)$ is

(A) even (B) odd (C) neither (D) unable to answer without being given $h(x)$ and $g(x)$ (E) N.O.T.A.

10. The equation $y^2 - 4x^2 = 0$ describes a degenerate conic section. What does the graph of this equation look like?

(A) a point (B) a line (C) two intersecting lines
(D) two parallel lines (E) N.O.T.A.

11. Which of the following best describes the graph of $x^2 + 2x - 9y^2 + y = 0$?

(A) line (B) parabola (C) circle (D) ellipse (E) hyperbola

12. Which of the following is a sixth root of -64?

(A) $\frac{\sqrt{3}}{2} + \frac{1}{2}i$ (B) $\sqrt{3} - i$ (C) $1 - \sqrt{3}i$ (D) $-1 - \sqrt{3}i$
(E) N.O.T.A.

13. Find the sum of the first 100 terms of the arithmetic sequence with $a_2 = -10$ and $a_{12} = 10$.

(A) 8500 (B) 8600 (C) 8700 (D) 8800 (E) N.O.T.A.

14. If $f(x) = x^2 + x$, find $\dfrac{f(x+h) - f(x)}{h}$, where $h \neq 0$.

(A) $2xh + h^2 + 1$ (B) h (C) 1 (D) $2x - h + 1$ (E) N.O.T.A.

15. The area of a rectangle is 20 ft^2. Express the perimeter $P(x)$ of this rectangle as a function of the length of one of its sides x.

(A) $P(x) = 2x + 2y$ (B) $P(x) = 2x + 2y - 20$
(C) $P(x) = 2x + 2(10 - x)$ (D) $P(x) = 2x + \frac{40}{x}$ (E) N.O.T.A.

16. What are the vertical asymptotes of the graph of $f(x) = \dfrac{x^2 - 4}{(x-5)(x+3)(x-2)}$?

(A) $x = 0$ (B) $x = 5$, $x = -3$, $x = 2$ (C) $x = -5$, $x = 3$, $x = -2$
(D) $x = 5$, $x = -3$ (E) N.O.T.A.

17. Find the sum of the infinite series $300 + 150 + \dfrac{200}{3} + \ldots + \dfrac{150 + 150k}{3^{k-1}} + \ldots$

(A) 900 (B) 550 (C) $\frac{1125}{2}$ (D) $\frac{1225}{3}$ (E) N.O.T.A.

18. The graph of $4x^2 + y^2 - 8x + 4y - 28 = 0$ is that of an ellipse. Find the length of the minor axis of the ellipse.

(A) 2 (B) 3 (C) 6 (D) $\sqrt{7}$ (E) N.O.T.A.

19. The graph of $y = f(x - 5) + 3$ is the graph of $y = f(x)$

(A) shifted to the left 5 units and up 3 units (B) shifted to the left 5 units and down 3 units (C) shifted to the right 5 units and up 3 units (D) shifted to the right 5 units and down 3 units (E) N.O.T.A.

20. Solve for x: $(x - 5)(x + 7) = 1$

(A) $5, -7$ (B) $-5, 7$ (C) $\pm\sqrt{37}$ (D) $\pm 2\sqrt{37}$ (E) N.O.T.A.

21. The graph of $y = x^2$ intersects the graph of $\frac{x^2}{4} + \frac{y^2}{36} = 1$ at two points. Find the distance between those points.

(A) $\sqrt{3}$ (B) $2\sqrt{3}$ (C) 6 (D) $4\sqrt{3}$ (E) 12

22. Suppose z is a complex number with $0 < |z| < 1$. Also suppose x and y are complex numbers such that $x = z^3$ and $z = y^3$. Which of the following is true?

(A) $|x| < |z| < |y|$ (B) $|y| < |z| < |x|$ (C) $|z| < |y| < |x|$
(D) $|z| < |x| < |y|$ (E) N.O.T.A.

23. Suppose that a polynomial $y = p(x)$ has the following characteristics:

(1) $p(x) \leq 0$ for $x \leq 5$
(2) $p(x) > 0$ for $x > 5$
(3) $f(-3) = f(1) = 0$

How many roots of odd multiplicity does $y = p(x)$ have?

(A) not enough information (B) 5 (C) 3 (D) 1 (E) N.O.T.A.

24. In the complex plane, the graph of $\{z : |z - 1 + i| = 2\}$ is a circle. What is the center and radius of this circle?

(A) center $1 + i$, radius 2 (B) center $1 - i$, radius 2
(C) center $1 + i$, radius $\sqrt{2}$ (D) center $1 - i$, radius $\sqrt{2}$
(E) N.O.T.A.

25. What common quantity must be added to each term of the sequence x, y, z in order to create a geometric sequence?

(A) $\dfrac{y^2 - xz}{z + x - 2y}$ (B) $\dfrac{xz + y^2}{x + z + 2y}$ (C) $\dfrac{y^2 - xz}{2y - x - z}$
(D) $\dfrac{y^2 + xz}{x - z - 2y}$ (E) N.O.T.A.

Tie Breakers

1. If $\frac{2}{3}$ is the eighth term of the geometric sequence $..., \frac{3}{2}, 1, \frac{2}{3}, ...$, then what is the first term?

2. What is the conjugate of $\frac{1+2i}{3-4i}$, written in $a + bi$ form?

3. How many solutions of $\sin^2\theta = -\cos^2\theta$ are there for $0 \leq \theta \leq 2\pi$?