Hoover High School Mathematics Tournament - March 5, 2005
Algebra 2 Ciphering

1.1 The 50th term of an arithmetic sequence is 200. If the common difference is 7, what is the sixth term?

A: -108

1.2 Let $f(x) = \dfrac{x+3}{x-7}$. Find $f^{-1}(-1)$.

A: 2

1.3 A hyperbola has vertices $(2,1)$ and $(2,3)$ and passes through the points $(1,0)$ and $(3,0)$. If the equation of this hyperbola is written as $a(y-k)^2 + b(x-h)^2 = 1$, find $a+k+b+h$.

A: 2

1.4 $\begin{vmatrix} x & 0 & 0 & 0 & x \\ 0 & x & 0 & x & 0 \\ 0 & 0 & x & 0 & 0 \\ 0 & x & 0 & x & 0 \\ x & 0 & 0 & 0 & x \end{vmatrix} = x^6$. Find the sum of all such x.

A: 0

1.5 Given perpendicular lines m and n, find the slope of line n, given that line m has negative slope, y-intercept b, and x-intercept a.

A: $\dfrac{a}{b}$

2.1 Solve for x: $\sqrt{(x-5)^2} = 10$

A: -5 and 15

2.2 If $g(x) = 3(x+5)^2$, find $\dfrac{g(x+h) - g(x)}{h}$, where $h \neq 0$.

A: $6x + 3h + 30$

2.3 Find all rational roots of the polynomial
$P(x) = x^5 + 10x^4 - 12x^3 - 266x^2 - 245x$.

A: 0, 5, -7, -1

2.4 An ellipse passes through the point $(2, 6\sqrt{2})$ and has the points $(6, 0)$ and $(-6, 0)$ as the endpoints of the minor axis. If the equation of this ellipse is written as $a(x-h)^2 + b(y-k)^2 = 1$, find $\dfrac{a+b}{ab}$.

A: 117

2.5 A coin is weighted with $P(\text{heads}) = 0.2$, $P(\text{tails}) = 0.1$, and $P(\text{landing on its edge}) = 0.7$. If you flip this coin six times, what is the probability of getting exactly 3 tails?

A: 0.01458 or $\dfrac{729}{50000}$

3.1 What are the vertical asymptotes of the function $y = \dfrac{x^2 - 4x}{x^2 - x - 12}$?

A: $x = -3$

3.2 Let A be the arithmetic mean and H be the harmonic mean of 6 and 12. What is the value of $\dfrac{A-H}{A+H}$?

A: $\dfrac{1}{17}$

3.3 Find the domain (in interval notation) of the function $f(x) = \sqrt{\dfrac{x^2 - 3x - 40}{x - 3}}$.

A: $[-5, 3) \cup [8, \infty)$

3.4 A parabola has focus $(0, 0)$ and directrix $x = 4$. If the equation of this parabola is written as $x = ay^2 + by + c$, find the arithmetic mean of a, b, and c.

A: $\dfrac{5}{8}$

3.5 Find the value of the expression $(\sum_{k=1}^{n} k)^2 \div (\sum_{k=1}^{n} k^3)$ when $n = 10$.

A: 1

4.1 A semi-elliptical arch over a tunnel through a mountain has a major

axis of 100 feet and a height at the center of 30 feet. Determine the height of the arch 5 feet from the center of the tunnel.

A: $9\sqrt{11}$ feet

4.2 Let z be a complex number. Find z if $2z + i = \overline{z} - 3i$.

A: $-\frac{4}{3}i$

4.3 Find all values of y that satisfy the equation $y^{\frac{2}{3}} + y^{\frac{1}{3}} = 6$.

A: 8 and -27

4.4 Find the value of $(1 - \sqrt{3}i)^{12}$, where $i = \sqrt{-1}$.

A: 4096

4.5 If $\dfrac{50}{(x+3)^2(x^2+1)}$ is decomposed into partial fractions, find the numerator of the fraction whose denominator is $x^2 + 1$.

A: $-3x + 4$

Extra 1 A sequence is defined recursively by the formula $a_{n+1} = \dfrac{3a_n}{a_{n-1}}$, where $a_1 = 2$ and $a_2 = 1$. What is the value of a_{2005}?

A: 2

Extra 2 If $x^{x^{x^{x^{\cdots}}}} = 2$, then $(((((x)^x)^x)^x)^x) = ?$

A: 4