## Hoover High School Mathematics Tournament - February 22, 2003

## Geometry Written Test

1. In the diagram, $\mathrm{AC} / / \mathrm{FD}$. What is the measure of $\angle F G E$ if $\mathrm{m} \angle A H B$ equals $126^{\circ}$ ?

a) $126^{\circ}$
b) $54^{\circ}$
c) $44^{\circ}$
d) $234^{\circ}$
e) NOTA
2. If an angle of a regular polygon equals $170^{\circ}$, how many sides does it have?
a) 32
b) 35
c) 36
d) 37
e) NOTA
3. Find the area of the shaded region in the following circle with center $O$.

a) $240 \pi$
b) $552 \pi$
c) $576 \pi$
d) $600 \pi$
e) NOTA
4. Large circle O circumscribes $\triangle A B C$ and small circle O is inscribed in $\triangle A B C$. Find the ratio of the sum of the radii to the positive difference of the sadii.

a) $\frac{1}{3}$
b) $\frac{1}{2}$
c) 2
d) 3
e) NOTA

5. If circle $O$ is inscribed in quadrilateral $A B C D$, what is $x$ ?
a) 6.5
b) 5.5
c) 4.5
d) 7.5
e) NOTA
6. For these circles with radius 6 and central angles of 60 degrees, what is the area of the shaded region?

a) $12 \pi-18 \sqrt{3}$
b) $36 \pi-36 \sqrt{3}$
c) $16 \pi+2 \sqrt{3}$
d) $5 \sqrt{3}-6 \pi$
e) NOTA
7. A right circular cylinder is inscribed in a sphere of radius 4 inches. The height of the cylinder is twice the radius of the cylinder. Find the volume of the cylinder.
a) $8 \pi \sqrt{3}$
b) $16 \pi \sqrt{2}$
c) $32 \pi \sqrt{2}$
d) $128 \pi$
e) NOTA
8. The arc of a sector has degree measure 60. The radius of the sector is 12 inches. Find the area of the circle that can be inscribed in the sector.
a) $3 \pi \sqrt{3}$
b) $16 \pi$
c) $27 \pi$
d) $48 \pi$
e) NOTA
9. Chords BD and CE are perpendicular. Express the area A of circle O in terms of $\mathrm{a}, \mathrm{b}$, $c$, and d.
a) $\frac{\pi(a-b)^{2}}{4}+\frac{\pi(c-d)^{2}}{4}$
b) $\pi\left(\frac{(b-c)^{2}}{4}+\frac{(a-d)^{2}}{4}\right)$
c) $\frac{\pi(a-c)^{2}}{4}+\frac{\pi(a-c)^{2}}{4}$
d) $\frac{\pi(a+c)^{2}}{4}+\frac{\pi(b+d)^{2}}{4}$
e) NOTA

10. A cone of height 14 is inscribed in a cylinder of lateral surface area $168 \pi$. Find the surface area of a sphere having the same radius as the cylinder.
a) $169 \pi$
b) $144 \pi$
c) $121 \pi$
d) $168 \pi$
e) NOTA
11. Find the area of a rhombus whose perimeter is 116 and sum of diagonals is 82 .
a) 164
b) 420
c) 840
d) 1680
e) NOTA
12. What is the area of a quadrilateral whose vertices arc located at $(2,5),(-4,10),(3,7)$, and $(-1,0)$ ?
a) 8
b) 23
c) 80
d) 31
e) NOTA
13. What is the measure of $x$ ?

a) $\frac{24}{7}$
b) $\frac{7}{24}$
c) 14
d) $\frac{14}{3}$
e) NOTA
14. What is the measure of angle $\theta$ ? ( $\theta$ is formed by a secant and a tangent)

a) 294
b) 49
c) 98
d) 66
e) NOTA
15. The common internal tangent of circles $\mathrm{O} \& \mathrm{P}$ is 17 . The radius of circle O is 7 , and the radius of circle $P$ is 5 . What is the distance between the points $O$ and $P$ ?

a) 48
b) $11 \sqrt{3}$
c) $\sqrt{301}$
d) $\sqrt{433}$
e) NOTA
16. A cone is cut horizontally into a smaller cone and a frustum having equal volume. If the larger cone has height $=12$ and radius $=4$, what is the volume of a sphere with radius equal to the smaller radius, $r$ ?

a) $\frac{8 \pi \sqrt{3}}{3}$
b) $\frac{4 \pi \sqrt{2}}{3}$
c) $\frac{32 \pi}{3}$
d) $\frac{128 \pi}{3}$
e) NOTA
17. A 6-8-10 right triangle has an altitude of length $A$ and median of length $M$ (both altitude and median are drawn to the hypotenuse). What is $\sqrt{M^{2}-A^{2}}$ ?
a) 1.3
b) 1.4
c) 1.69
d) 1.96
e) NOTA
18. Find the distance between the point $(2002,2003)$ and the line $3 x-2 y=0$.
a) $\frac{2000 \sqrt{13}}{13}$
b) $\frac{2001 \sqrt{13}}{13}$
c) $\frac{2002 \sqrt{13}}{13}$
d) $\frac{2003 \sqrt{13}}{13}$
e) NOTA
19. If radius of circle E is 3 and radius of circle F is 6 , then $\mathrm{BD}:=$ ?

a) 24
b) $18+3 \sqrt{3}$
c) $18+6 \sqrt{3}$
d) 27
e) NOTA
20. A cylindrical tank 20 feet long and 8 feet in diameter is placed so that its axis is horizontal. When the tank is filled to a depth of 2 feet, how many cubic feet of liquid does it contain?
a) $\frac{320 \pi}{3}-80 \sqrt{3}$
b) $64-12 \sqrt{7}$
c) $\frac{16 \pi}{3}-4 \sqrt{3}$
d) $\frac{160 \pi}{3}-80 \sqrt{3}$
e) NOTA
21. Trapezoid ABCD is isosceles. If $\mathrm{AB}=4, \mathrm{DC}=8$, and the area of $\triangle \mathrm{CDF}=1360$, what is the area of the trapezoid?

a) 1530
b) 3060
c) 2030
d) 2003
e) NOTA
22. 

$\angle A=120^{\circ}$
$\overline{A B}=\overline{A C}=10 ; \mathrm{D}$ and E are midpoints What is the area of DEFG?

a) $\frac{25}{9}$
b) 100
$\begin{array}{ll}\text { d) } \frac{75}{4} & \text { e) NOTA }\end{array}$
c) $\frac{25 \sqrt{3}}{2}$
23. Henry, Irving, and Weichen have congruent, externally tangent, circular yards of radius 20 as shown. They want to build a circular kiddy pool whose rim will intersect points of tangency $\mathrm{H}, \mathrm{I}$ and W . If the height of the pool is 3 , what will its volume be?

a) $\frac{400}{3} \pi$
b) $1200 \pi$
c) $400 \pi$
d) $3600 \pi$
e) NOTA
24. The measure of $\angle P A Q$ is $60^{\circ}, \overrightarrow{A B}$ bisects $\angle P A Q$ and circles P and Q are tangent to $\overrightarrow{A B}$. If the radii of circies P and Q are 1 and 2 respectively, compute the distance from $P$ to $Q$.
a) 1
b) $\sqrt{3}$
c) 2
d) $2 \sqrt{3}$
c) NOTA
25.


ABCD is a square with side length 10 inscribed in circle O . Square EFGH is inscribed in the segment cut off by chord CD. Find the arec of square EFGH.
a) 4
b) $75-50 \sqrt{2}$
c) $50-10 \sqrt{2}$
d) $25+15 \sqrt{2}$
e) NOTA
$T_{1}$ ) How many diagonals are there in a 2003-gon?
$\mathrm{T}_{2}$ ) What is the volume of a regular hexahedron with side lengtl $1.5 \times 10^{4}$ ?
$\mathrm{T}_{3}$ ) The sides of a triangle are 5,5, and 6 inches long. Find the ratio of the area of a circle inscribed in the triangle to the area of the circle circumscribing the triangle.

