Hoover High School Mathematics Tournament – February 22, 2003 Geometry Written Test

1. In the diagram, AC // FD. What is the measure of $\angle FGE$ if m $\angle AHB$ equals 126°?



2. If an angle of a regular polygon equals 170°, how many sides does it have?

- a) 32 b) 35 c) 36 d) 37 e) NOTA
- 3. Find the area of the shaded region in the following circle with center O.



4. Large circle O circumscribes $\triangle ABC$ and small circle O is inscribed in $\triangle ABC$. Find the ratio of the sum of the radii to the positive difference of the radii.



6. For these circles with radius 6 and central angles of 60 degrees, what is the area of the shaded region?



- a) $12\pi 18\sqrt{3}$ b) $36\pi 36\sqrt{3}$ c) $16\pi + 2\sqrt{3}$ d) $6\sqrt{3} 6\pi$ e) NOTA
- 7. A right circular cylinder is inscribed in a sphere of radius 4 inches. The height of the cylinder is twice the radius of the cylinder. Find the volume of the cylinder.
 - a) $8\pi\sqrt{3}$ b) $16\pi\sqrt{2}$ c) $32\pi\sqrt{2}$ d) 128π e) NOTA
- 8. The arc of a sector has degree measure 60. The radius of the sector is 12 inches. Find the area of the circle that can be inscribed in the sector.
 - a) $3\pi\sqrt{3}$ b) 16π c) 27π d) 48π e) NOTA
- Chords BD and CE are perpendicular. Express the area A of circle O in terms of a, b, c, and d.



10. A cone of height 14 is inscribed in a cylinder of lateral surface area 168π . Find the surface area of a sphere having the same radius as the cylinder.

a) 169π b) 144π c) 121π d) 168π e) NOTA

- 11. Find the area of a rhombus whose perimeter is 116 and sum of diagonals is 82.
 - a) 164 b) 420 c) 840 d) 1680 e) NOTA
- 12. What is the area of a quadrilateral whose vertices arc located at (2,5), (-4,10), (3,7), and (-1,0)?
 - a) 8 b) 23 c) 80 d) 31 e) NOTA
- 13. What is the measure of x?



14. What is the measure of angle θ ? (θ is formed by a secant and a tangent)



15. The common internal tangent of circles O & P is 17. The radius of circle O is 7, and the radius of circle P is 5. What is the distance between the points O and P?



16. A cone is cut horizontally into a smaller cone and a frustum having equal volume. If the larger cone has height = 12 and radius = 4, what is the volume of a sphere with radius equal to the smaller radius, r?



a)
$$\frac{8\pi\sqrt{3}}{3}$$
 b) $\frac{4\pi\sqrt{2}}{3}$ c) $\frac{32\pi}{3}$ d) $\frac{128\pi}{3}$ e) NOTA

- 17. A 6-8-10 right triangle has an altitude of length A and median of length M (both altitude and median are drawn to the hypotenuse). What is $\sqrt{M^2 A^2}$?
 - a) 1.3 b) 1.4 c) 1.69 d) 1.96 e) NOTA
- 18. Find the distance between the point (2002,2003) and the line 3x 2y = 0.
 - a) $\frac{2000\sqrt{13}}{13}$ b) $\frac{2001\sqrt{13}}{13}$ c) $\frac{2002\sqrt{13}}{13}$ d) $\frac{2003\sqrt{13}}{13}$ e) NOTA

19. If radius of circle E is 3 and radius of circle F is 6, then BD = ?



- a) 24 b) $18 + 3\sqrt{3}$ c) $18 + 6\sqrt{3}$ d) 27 e) NOTA
- 20. A cylindrical tank 20 feet long and 8 feet in diameter is placed so that its axis is horizontal. When the tank is filled to a depth of 2 feet, how many cubic feet of liquid does it contain?

a)
$$\frac{320\pi}{3} - 80\sqrt{3}$$
 b) $64 - 12\sqrt{7}$ c) $\frac{16\pi}{3} - 4\sqrt{3}$ d) $\frac{160\pi}{3} - 80\sqrt{3}$ e) NOTA

21. Trapezoid ABCD is isosceles. If AB = 4, DC = 8, and the area of \triangle CDE = 1360, what is the area of the trapezoid?



23. Henry, Irving, and Weichen have congruent, externally tangent, circular yards of radius 20 as shown. They want to build a circular kiddy pool whose rim will intersect points of tangency H, I and W. If the height of the pool is 3, what will its volume be?



24. The measure of $\angle PAQ$ is 60°. \overrightarrow{AB} bisects $\angle PAQ$ and circles P and Q are tangent to \overrightarrow{AB} . If the radii of circles P and Q are 1 and 2 respectively, compute the distance from P to Q.



25.



ABCD is a square with side length 10 inscribed in circle O. Square EFGH is inscribed in the segment cut off by chord CD. Find the area of square EFGH.

- a) 4 b) $75-50\sqrt{2}$ c) $50-10\sqrt{2}$ d) $25+15\sqrt{2}$ e) NOTA
- T₁) How many diagonals are there in a 2003-gon?
- T₂) What is the volume of a regular hexahedron with side length $1.5 \ge 10^4$?
- T_3) The sides of a triangle are 5, 5, and 6 inches long. Find the ratio of the area of a circle inscribed in the triangle to the area of the circle circumscribing the triangle.