Hoover High School Mathematics Tournament - February 22, 2003 Comprehensive Test

1. $\frac{x-13}{x^2+4x-5} = \frac{A}{x+B} - \frac{C}{x-D}$, where A, B, C, D are relatively prime positive integers, find $A^4 + B^3 + C^2 + D$.

a) 211 b) 221 · c) 39 d) 32 e) NOTA

2. Find the x-intercept of graph:
$$\begin{cases} x = 10 - \frac{5}{3}t \\ y = 25t^2 + 90t + 81 \end{cases}$$

a) 7 b) 13 c) 441 d) 1521 e) NOTA

3. Find the length of the interval for x on which $\frac{9x+18}{x^2-4} < -3$

a) 6 b) 5 c) 3 d) interval is infinite e) NOTA

4. Given:
$$\begin{cases} x + y = 8 \\ xy = 4 \end{cases}$$
, find $x^3 + y^3$.
a) 56 b) 416 c) 480 d) 512 e) NOTA

5. Solve for x over
$$\Re$$
: $\sqrt{x^{x^{x^*}}} \sqrt{x^{x^{x^*}}} \sqrt{x^{x^{x^*}}} = 5$

a) $\frac{\sqrt{5}}{5}$ b) $\sqrt[3]{5}$ c) $\sqrt{5}$ d) 5 e) NOTA

6. If $|\cos\theta + \sin\theta| = 1$, how many angles θ exist on the domain $[0, 2\pi]$?

a) 3
b) 4
c) 5
d) infinitely many e) NOTA
7. In how many distinguishable ways can the letters in the word "ERINNGIOIA" be arranged?
a) 302,400
b) 604,800
c) 1,814,400
d) 3,628,800
e) NOTA

- 8. The sum of Q quarters and D dollar bills in Patrick's wallet is \$82.50. Given that (Q,D) is a random set of two whole numbers, find the probability that $Q \ge D$.
- a) $\frac{17}{83}$ b) $\frac{16}{83}$ c) $\frac{66}{83}$ d) $\frac{67}{83}$ e) NOTA
- 9. The equation 2x + 3y = 7 has *n* solutions (x, y) such that x and y are positive integers. Find $\left[\sqrt{17n} 2\right]$ where [m] is the greatest integer less than or equal to *m*.
- a) 2 b) 3 c) 4 d) 15 e)NOTA
- 10. If $f(2x, y-3) = \frac{xy+4y+1}{3x+1}$, where $x \neq \frac{1}{3}$, find f(6,2).
- a) $\frac{-21}{17}$ b) $\frac{-3}{7}$ c) $\frac{9}{2}$ d) $\frac{18}{5}$ e)NOTA

11. If $y = (-19)(-18)(-17)(-16)(-15)\cdots(8)(9)(10)(11)$, then which of the following is true for y?

- a) |y| > y b) $\frac{|y|}{y} = -1$ c) $\sqrt{y^2} = -y$ d) 4y < y e) all are true
- 12. Josh and Mary are playing a game. In the game, players take turns rolling two dice. The first player to roll doubles or a sum of eight wins, and the game ends. If Josh rolls first, what is the probability that he wins on his second roll?
- a) $\frac{25}{216}$ b) $\frac{64}{729}$ c) $\frac{845}{5832}$ d) $\frac{6875}{46656}$ e)NOTA
- 13. Given the following triangles, find y^2 in terms of x.





15. Given: $\begin{cases} f(x) = (x-3)^4 \\ g(x) = (x-1)^2 \end{cases}$. Find the probability that $f(x) \le g(x)$, if x is a random variable on the

interval [0,6].

- a) $\frac{1}{4}$ b) $\frac{1}{3}$ c) $\frac{1}{2}$ d) $\frac{2}{3}$ e) NOTA
- 16. When Wasef and Nadeem are stacking books, they notice that a piece of 12 in by 18 in. cardboard has four squares of side length x cut out of its corners. Folding up the flaps, Wasef makes an open box with height x in³. Nadeem notes that the volume of the box is 160. Find the sum of the possible values for x.

a) 4 b) 5 c) 10 d) 15 e) NOTA

17. If $a_n = \frac{2}{(n+4)(n+5)}$ for all positive integers *n*, find the value *n* such that $\frac{a_n}{a_{n-1}} = \frac{4}{5}$.

a) 5 b) 6 c) 7 d) 8 e)NOTA

18. If:

$$a = \text{the remainder in: } \frac{999500}{2003} ,$$

$$b = \left[\log \left(\frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \frac{1}{10000} + \dots \right) \right], \text{ where } \left[\left[\right] \text{ is the greatest integer function,} \right]$$

$$c = \sqrt{5 + \sqrt{5 + \sqrt{121}}} ,$$

find the sum of the sum of the roots and the product of the roots in the equation $ax^2+bx+c=0$.

a)
$$\frac{1}{2}$$
 b) $\frac{2}{3}$ c) 1 d) $\frac{4}{3}$ e) NOTA

 $b = e^{-m} - (\cos 0 + i \sin 0)^3$ $c = \log_{\left(\frac{5}{6}\right)} \frac{7776}{3125}$ Find: $\frac{a^1}{c+a}$

a) $\frac{-3}{5}$ b) $\frac{-2}{5}$ c) $\frac{-1}{5}$ d) $\frac{2}{5}$ e)NOTA

23.	3. Evaluate the remainder:		$\frac{12!+11!+10!}{13}$		
a)	3	b) 6	c) 9	d) 12	e)NOTA

24. A circle is inscribed in an ellipse, which is inscribed in a larger circle, each having a common center. If the equation of the ellipse is $3x^2 + 75y^2 - 300 = 0$, and the ratio of the area of the shaded region to the

area of the unshaded region is $\frac{m}{n}$, where m and n are relatively prime integers, find m-n.



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a) -12 b) -17 c) -21 d) -25 e)NOTA

25. Find the numerical value of sec 20° sec 40' sec 80°

a)
$$\frac{1}{5}$$
 b) $\frac{\sqrt{6} + \sqrt{3}}{4}$ c) 5 d) 8 e)NOTA

TB1 Evaluate:
$$\log_2 \left(\sin \left(\frac{3\pi}{8} + \frac{3\pi}{16} + \frac{3\pi}{32} + \dots + \frac{3\pi}{8(2^{n-1})} + \dots \right) \right)$$

TB2 How many line segments are in this figure?



TB3 In decimal form, $\frac{400}{81}$ =4.abcde1605..., where a, b, c, d, and e each represent a digit. Find $\frac{b+e}{c+d}$ + a