## Hoover High School Mathematics Tournament - February 22, 2003 Comprehensive Test

1. $\frac{x-13}{x^{2}+4 x-5}=\frac{A}{x+B}-\frac{C}{x-D}$, where $A, B, C, D$ are relatively prime positive integers, find $A^{4}+B^{3}+C^{2}+D$
a) 211
b) 221
c) 39
d) 32
e) NOTA
2. Find the x-intercept of graph: $\left\{\begin{array}{l}x=10-\frac{5}{3} t \\ y=25 t^{2}+90 t+81\end{array}\right.$
a) 7
b) 13
c) 441
d) $152: 1$
e) NOTA
3. Find the length of the interval for x on which $\frac{9 x+18}{x^{2}-4}<-3$
a) 6
b) 5
c) 3
d) interval is infinite
e) NOTA
4. Given: $\left\{\begin{array}{c}x+y=8 \\ x y=4\end{array}\right.$, find $x^{3}+y^{3}$.
a) 56
b) 416
c) 480
d) 512
e) NOTA

a) $\frac{\sqrt{5}}{5}$
b) $\sqrt[3]{5}$
c) $\sqrt{5}$
d) 5
e) NOTA
5. If $|\cos \theta+\sin \theta|=1$, how many angles $\theta$ exist on the domain $[0,2 \pi]$ ?
a) 3
b) 4
c) 5
d) infinitely many
e) NOTA
6. In how many distinguishable ways can the letters in the word "ERINNGIOIA" be arranged?
a) 302,400
b) 604,800
c) $1,814,400$
d) $3,62,8,800$
e) NOTA
7. The sum of Q quarters and D dollar bills in Patrick's wallet is $\$ 82.50$. Given that ( $\mathrm{Q}, \mathrm{D}$ ) is a random set of two whole numbers, find the probability that $\mathrm{Q}>\mathrm{D}$.
a) $\frac{17}{83}$
b) $\frac{16}{83}$
c) $\frac{66}{83}$
d) $\frac{67}{83}$
e) NOTA
8. The equation $2 x+3 y=7$ has $n$ solutions $(x, y)$ such that $x$ and $y$ are positive integers. Find $[\sqrt{17 n}-2]$ where $[m]$ is the greatest integer less than or equal to $m$.
a) 2
b) 3
c) 4
d) 15
e)NOTA
9. If $f(2 x, y-3)=\frac{x y+4 y+1}{3 x-1}$, where $x \neq \frac{1}{3}$, find $f(6,2)$.
a) $\frac{-21}{17}$
b) $\frac{-3}{7}$
c) $\frac{9}{2}$
d) $\frac{18}{5}$
e) NOTA
10. If $y=(-19)(-18)(-17)(-16)(-15) \cdots(8)(9)(10)(1 \mathrm{I})$, then which of the following is true for $y$ ?
a) $|y|>y$
b) $\quad \frac{|y|}{y}=-1$
c) $\sqrt{y^{2}}=-y$
d) $4 y<y$
e)all are true
11. Josh and Mary are playing a game. In the game, players take turns rolling two dice. The first player to roll doubles or a sum of eight wins, and the game ends. If Josh rolls first, what is the probability that he wins on his second roll?
a) $\frac{25}{216}$
b) $\frac{64}{729}$
c) $\frac{845}{5832}$
d) $\frac{6875}{46656}$
e) NOTA
12. Given the following triangles, find $y^{2}$ in terms of $x$.

13. Find $x^{\prime \prime}$. Given:
$A B:=4, B C=8, A C=9, y=A D, x=C D$ and $\overline{B D}$ is an angle bisector

a) 18
b) 144
c) 216
d) 729
e) NOTA
14. Given $\left\{\begin{array}{l}f(x)=(x-3)^{4} \\ g(x)=(x-1)^{2}\end{array}\right.$. Find the probability that $\mathrm{f}(\mathrm{x}) \leq \mathrm{g}(\mathrm{x})$, if x is a random variable on the interval $[0,6]$.
a) $\frac{1}{4}$
b) $\frac{1}{3}$
c) $\frac{1}{2}$
d) $\frac{2}{3}$
e) NOTA
15. When Wasef and Nadeem are stacking books, they notice that a piece of 12 in . by 18 in . cardboard has four squares of side length $x$ cut out of its corners. Folding up the flaps, Wasef makes an open box with height $x \mathrm{in}^{3}$. Nadeem notes that the volume of the box is 160 . Find the sum of the possible values for $x$.
a) 4
b) 5
c) 10
d) 15
e) NOTA
16. If $a_{n}=\frac{2}{(n+4)(n+5)}$ for all positive integers $n$, find the value $n$ such that $\frac{a_{n}}{a_{n 1}}=\frac{4}{5}$
a) 5
b) 6
c) 7
d) 8
e)NOTA
17. If: $a=$ the remainder in: $\frac{999500}{2003}$,

$$
\begin{aligned}
& b=\left\|\log \left(\frac{1}{10}+\frac{1}{100}+\frac{1}{1000}+\frac{1}{10000}+\ldots\right)\right\|, \text { where } \llbracket \rrbracket \text { is the greatest integer function, } \\
& c=\sqrt{5+\sqrt{5+\sqrt{121}}}
\end{aligned}
$$

find the sum of the sum of the roots and the product of the roots in the equation $a x^{2}+b x+c=0$.
a) $\frac{1}{2}$
b) $\frac{2}{3}$
c) 1
d) $\frac{4}{3}$
e) NOTA
19. Evaluate: $\lim _{x \rightarrow 1} \frac{\sqrt[3]{3 x+1}}{\sqrt[3]{x-2}}$
a) $\sqrt[3]{3}$
b) $\sqrt{3}$
c) 3
d) $\infty$
e) NOTA
20. Evaluate: $\left|\begin{array}{ccc}\left|\begin{array}{ccc}5 & 5 & 4 \\ 3 & 2 & 1 \\ -9 & -6 & -3\end{array}\right|\left|\begin{array}{ccc}15 & 6 & -1 \\ 4 & 3 & -1 \\ -5 & 0 & 0 \\ 11 & 10 & 2\end{array}\right| \\ -2 & -7 & 1 \\ 4 & 5 & 0\end{array}\right|\left|\begin{array}{ccc}0 & 0 & 8 \\ 0 & 4 & 19 \\ 1 & 5 & -15\end{array}\right|$
a) -2892
b) 2860
c) 2828
d) 2892
e) NOTA
21. Evaluate $\sum_{n=1}^{\infty} \frac{2^{n+1} 3^{2 n-2}}{24^{n}}$
a) no sum
b) 4
c) 16
d) $\frac{64}{3}$
e) NOTA
22. Given: $a=$ the dot product between $\langle 3,4,-2\rangle$ and vector $\bar{t}$, where $\bar{t}$ is $\langle 3,4,-2\rangle \times\langle 1,-1,0\rangle$

$$
\begin{aligned}
& b-e^{-m}-(\cos 0+i \sin 0)^{3} \\
& c=\log _{\left[\left.\frac{3}{6} \right\rvert\,\right.} \frac{7776}{3125}
\end{aligned}
$$

Find $\frac{a^{\prime}}{c+a}$
a) $\frac{-3}{5}$
b) $\frac{-2}{5}$
c) $\frac{-1}{5}$
d) $\frac{2}{5}$
e)NOTA
23. Evaluate the remainder: $\frac{12!+11!+10!}{13}$
a) 3
b) 6
c) 9
d) 12
e) NOTA
24. A circle is inscribed in an ellipse, which is inscribed in a larger circle, each having a common center. If the equation of the ellipse is $3 x^{2}+75 y^{2}-300=0$, and the ratio of the area of the shaded region to the area of the unshaded region is $\frac{m}{n}$, where $m$ and $n$ are relatively prime integers, find $m-n$.

a) -12
b) -17
c) -21
d) -25
e)NOTA
25. Find the numerical value of $\sec 20^{\circ} \sec 40^{\circ} \sec 80^{\circ}$
a) $\frac{1}{5}$
b) $\frac{\sqrt{6}+\sqrt{3}}{4}$
c) 5
d) 8
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TBI Evaluate: $\log _{2}\left(\sin \left(\frac{3 \pi}{8}+\frac{3 \pi}{16}+\frac{3 \pi}{32}+\cdots+\frac{3 \pi}{8\left(2^{x-1}\right)} \div \cdots\right)\right)$

TB2 How many line segments are in this figure?


TB3 In decimal form, $\frac{400}{81}=4$ abcde $1605 \ldots$, where a, b, c, d, and e each represent a digit. Find

$$
\frac{b-e}{c+d}+a
$$

