Hoover High School Mathematics Tournament - February 22, 2003
Algebra 2 Test

1. Find the sum of the solutions to the equations: $x^{\left(3x^2+17x-96\right)}=1$

   a) $\frac{20}{3}$ b) 1 c) $\frac{-14}{3}$ d) $\frac{-17}{3}$ e)NOTA

2. Find the sum: $\frac{\pi}{e^2}+\frac{\pi^2}{e^4}+\frac{\pi^3}{e^6}+\cdots\frac{\pi^n}{e^{2n}}\cdots$

   a) $\frac{\pi}{e^2-\pi}$ b) $\frac{e^2}{e^2-\pi}$ c) $\frac{\pi}{e^2-e\pi}$ d) $\frac{e}{e-\pi}$ e)NOTA

3. Simplify: $1500i^{25}+500i^{36}+2i^{222}+i^{2003}$

   a) $2003i$ b) $498-1501i$ c) $498+1499i$ d) $-502+1499i$ e)NOTA

4. x varies directly with the square of y and inversely with the square root of z. If $x=4$ and $y=4$ when $z=8$, find y^2 when $x=2$ and $z=16$.

   a) $4\sqrt{2}$ b) 4 c) $2\sqrt[4]{8}$ d) $8\sqrt{2}$ e)NOTA

5. $5\log_3 2+2\log_9 10=?$

   a) $10\log_9 40$ b) $\log_9 50$ c) $\log_3 100$ d) $\log_3 320$ e)NOTA

6. Given: $f(x)=2x+f(x-1)$ and $f(1)=4$, find: $f(9)$

   a) 26 b) 36 c) 50 d) 92 e)NOTA

7. The 3rd term of a geometric sequence is -4, and the 1st term is $\frac{-16}{25}$. Find the 5th term.

   a) $\frac{-216}{25}$ b) $\frac{-25}{4}$ c) -100 d) -25
   e)NOTA

8. Pyry is bored, so he spends a day flipping a fair two-sided coin. If his first three flips were three consecutive heads, what is the probability of the next three flips containing exactly one head?

a) $\frac{1}{8}$ b) $\frac{3}{8}$ c) $\frac{3}{64}$ d) $\frac{15}{64}$ e)NOTA

9. On Wednesdays, Yang can buy hamburgers at McDonald's for 29 cents each. One Wednesday Yang had 9 more nickels than quarters. The number of quarters he had was one more than three times the number of pennies he had. He had a total of 39 coins. What was the maximum number of hamburgers that he could buy that day?

a) 14 b) 15 c) 16 d) 17 e)NOTA

10. A, B, C, and D are roots of the equation $3x^4 - 36x^3 + 73x^2 - 2003x + 9 = 0$. Find $A^2B^2C^2D^3 + A^2B^2C^3D^2 + A^2B^3C^2D^2 + A^3B^2C^2D^2$.

a) 108 b) 64 c) 36 d) –64 e)NOTA

11. Evaluate: $\sqrt{49\sqrt{49\sqrt{49\sqrt{\cdots}}}} - \sqrt[3]{64\sqrt[3]{64\sqrt[3]{64\sqrt[3]{\cdots}}}}$

a) 41 b) 33 c) 17 d) -15 e)NOTA

12. If the conjugate of the reciprocal of $\frac{5-3i}{20+21i}$ is written in the form $\frac{a+bi}{c}$, where $|a|, |b|$, and $c|$ are relatively prime and $b < 0$, find $\frac{b}{c-a}$.

a) $\frac{37}{131}$ b) $\frac{131}{37}$ c) $\frac{1}{55}$ d) 55 e)NOTA

13. Find the sum of the units digits of 2^{2003} and 5^{2002}.

a) 7 b) 9 c) 11 d) 13 e)NOTA

14. Solve for x: $11_{5x} + 22_{6x} = 122_{10}$

a) 4 b) 5 c) 6 d) 7 e)NOTA

15. Evaluate: $\sum_{x=1}^{5} x^2 + \sum_{x=1}^{20} 6x - \sum_{x=2}^{20} 3x$

a) 685 b) 688 c) 1975 d) 1942 e)NOTA

16. Evaluate the determinant: $\begin{vmatrix} \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ -2 & -1 \end{vmatrix} & \begin{vmatrix} 3 & 0 \\ 0 & 2 \end{vmatrix} \\ \begin{vmatrix} 0 & 2 \\ -2 & 1 \end{vmatrix} & \begin{vmatrix} 4 & 2 \\ 2 & 2 \end{vmatrix} & \begin{vmatrix} 4 & -3 \\ 2 & -2 \end{vmatrix} \\ \begin{vmatrix} 1 & 2 \\ -2 & 3 \end{vmatrix} & \begin{vmatrix} 0 & 0 \\ 4 & 4 \end{vmatrix} & \begin{vmatrix} 18 & 6 \\ 1 & \frac{1}{2} \end{vmatrix} \end{vmatrix}$

a) -170 b) -158 c) -142 d) 218 e)NOTA

17. If p is a positive constant where the roots of the equation $x^2 - 4px - 4x + 4 = 0$ are p and $9p$, find the numerical value of the sum of the roots of the equation.

a) $\frac{2}{3}$ b) 4 c) $\frac{20}{3}$ d) 10 e)NOTA

18. If ${}_nC_{(n-1)} + {}_{(n+1)}C_n = 5$ and $2^n(2^{2n}) = 4^3$, find n^n.

a) 16 b) 9 c) 8 d) 32 e)NOTA

19. The equation of the parabola containing the points $(1,-23),(2,18),(3,34)$ can be written as $y = ax^2 + bx + c$, where $|a|, |b|$, and $|c|$ are relatively prime. Find $a+b+c$.

a) -180 b) -23 c) 2 d) 180 e)NOTA

20. Find the sum of all values of a for which $\sqrt{a+\sqrt{a-4}} = 2$.

a) 9 b) 5 c) 4 d) 1 e)NOTA

21. Find $a+b$, given $\begin{matrix} |a+bi| = 5 \\ |4a+5i| = 13 \\ |2b+15i| = 17 \end{matrix}$, where a and b are positive.

a) 7 b) 12 c) 30 d) 56 e)NOTA

22. Let $(h,k)=$ the center of the circle $2x^2+2y^2+20x-4y+45=0$

Let $2a=$ the length of the major axis of the ellipse $13x^2+16y^2=208$

Let $y=mx+b$ be an asymptote of the hyperbola $16y^2-25x^2=400$

Find: $h+k+a^2+b^2$

a) 9 b) 12 c) 17 d) 20 e)NOTA

23. Given: $\frac{\log_b a}{\log_c a}=\frac{2003}{2002}$ and $\frac{b}{c}=c^k$. Solve for k.

a) $\frac{1}{2003}$ b) $\frac{1}{2002}$ c) $\frac{-1}{2003}$ d) $\frac{-1}{2002}$ e)NOTA

24. If the fourth term in the expansion of $(2x+5)^n$ is $5,280,000x^a$, where a and n are positive integers, find $n+a$.

a) 11 b) 12 c) 19 d) 20 e)NOTA

25. If $a_n=\frac{4}{(n+2)(n+3)}$ for all positive integers n, find the value of n such that $\frac{a_n}{a_{n-2}}=\frac{2}{5}$.

a) 2 b) 3 c) 4 d) 5 e)NOTA

Tiebreaker 1: Find x so that $x+4$, $x-4$, $x-5$, make a Pythagorean triple. (Note: $x+4, x-4, x-5$ are each positive)

Tiebreaker 2: Simplify: $e^{\left(\frac{\log_7 \frac{9}{5}}{2\log_{49} e}\right)}$

Tiebreaker 3: In how many distinguishable ways can Bart permute the letters of the word "KRABAPPLE"?