Hoover High School Mathematics Tournament – February 22, 2003 Algebra I Written Test

1. Find the sum of all values of x if: $\frac{2x+2}{6} = \frac{3}{x-1}$ $c)\sqrt{10}$ a) -10 b) 0 d) 10 e) NOTA 2. Find the area of the closed region bounded by the graph of |x| + |y| = 2b) 6 a) 8 c)4 d) 2 e) NOTA 3. Simplify: $-2\left[\frac{1}{5}(10x+3)+\left(-\frac{2}{5}\right)\right]$ a) $-4x + \frac{4}{5}$ b) $4x + \frac{2}{5}$ c) $-4x - \frac{2}{5}$ d) $-20x + \frac{26}{5}$ e) NOTA 4. Solve for x: $7(x+1)+2x^2 = 12-2x(5-x)$ a) $\frac{5}{17}$ b) $\frac{17}{5}$ c) $-\frac{5}{17}$ d) $-\frac{17}{5}$ e) NOTA 5. Simplify: $\frac{ax^2 - ay^2}{y - r}$ c) -a(x+y) d) already in simplest form a) a(y-x)b) a(x-y) e) NOTA 6. Solve for n: $81^3 = 3^{(n^2)}$ a) $\pm \sqrt{3}$ b) $+\sqrt{5}$ c) $+2\sqrt{3}$ d) $+2\sqrt{5}$ e) NOTA 7. The equation of a line that passes through the point (0,5) and has slope 2 is Ax + By = C, where A<B<C and |A|, |B|, and |C| are relatively prime. Find A+B+C. b) 4 a) -2 c) 7 d) 8 e) NOTA 8. Simplify: $\left[\frac{2xy^{-2} \cdot y^4}{3yx^{-1}}\right]^{-2} \cdot \left[\frac{4xy}{2x^{-1}y^3}\right]^2$

a)
$$9y^{10}$$
 b) $\frac{6}{y^6}$ c) $\frac{8}{3y^6}$ d) $\frac{9}{y^6}$ e) NOTA

- 9. How much 80% acid solution (in grams) must be added to a 125g sample of 10% acid to produce 50% solution?
- a) 170g b) $\frac{500}{3}$ g c) 75g d) $\frac{375}{4}$ g e) NOTA
- 10. If y varies directly with x and inversely with z^2 and y = 15 when x = 8 and z = 2, find y when x = 3 and z = 3.
- a) $\frac{21}{50}$ b) $\frac{9}{10}$ c) $\frac{18}{5}$ d) $\frac{45}{2}$ e) NOTA

11. If $x(5x-4)(x+3)(7x-2) = Ax^4 + Bx^3 + Cx^2 + Dx + E$, find A+B+C+D+E.

a) 23 b) 22 c) 21 d) 20 e) NOTA

12. Simplify:
$$5 + \frac{1}{5 + \frac{1}{5 + \frac{1}{5 + \frac{1}{5 + \frac{1}{5}}}}}$$

a) $\frac{18901}{3640}$ b) $\frac{3640}{701}$ c) $\frac{701}{135}$ d) $\frac{135}{26}$ e) NOTA

c) 25

13. Simplify: $5(5^{(\pi+1)})^{(\pi-1)}$

b) 5"

a) $5^{(\pi^2)}$

d)
$$25^{\pi}$$
 e) NOTA

14. Simplify:

$$\frac{\left[\frac{(x-2)^{2}(x+1)}{(x-2)^{0}(x^{2}+8x+15)}\right]}{\left[\frac{x^{3}-x-2}{x^{2}-x-30}\right]}$$
a) 1 b) $\frac{x-6}{x-3}$ c) $\frac{x^{2}-8x+12}{x+3}$ d) $\frac{(x-2)^{2}(x+1)^{2}}{(x+5)^{2}(x+3)(x-6)}$

15. Factor completely: $x^{5} - 6x^{4} - 24x^{3} + 134x^{2} - 105x$

a)
$$x(x-1)(x-3)(x-5)(x+7)$$

b) $x(x+1)(x+3)(x-5)(x+7)$
c) $x(x+1)(x-3)(x-5)(x+7)$
d) $x(x-1)(x-3)(x+5)(x-7)$
e) NOTA

16 If 3 is a root of the equation $x^2 - px + 2p + 1 = 0$, find the other root.

a) -1 b) 1 c) -7 d) 7 e) NOTA

17. Find the distance between the two intersections of the functions: $\frac{f(x) = x^2 - 2}{g(x) = x}$

a) 3 b)
$$\sqrt{2}$$
 c) 2 d) $3\sqrt{2}$ e) NOTA

18. If $a^*b = \frac{ab^2}{b-a^2}$ and $2^*x = \frac{-9}{11}$, find x if x < 0. a) $\frac{-12}{11}$ b) $\frac{-36}{31}$ c) $\frac{-3}{2}$ d) $\frac{-9}{22}$ e) NOTA

19. Find the length of the line 8x - 15y = 30 between x = -2 and x = 28.

20. If
$$\frac{x!}{7!} = 8$$
, solve for x.
a) 7 b) -8 c) 8 d) -7 e) NOTA

21 Which of the following expressions are equivalent when x > y > 0?

1.
$$\frac{x^4 - y^4}{x^2 + y^2}$$
 II. $x^2 - y^2$ III. $(x - y)\sqrt{x(x + 2y) + y^2}$ IV. $-(x + y)^2 + 2xy - 2x^2$

a) I, III, IV only b) I and II only c) II, III, IV only d) I, II, III only e) NOTA

22. What is $2.5\overline{2}$ expressed as a fraction in lowest terms?

a)
$$\frac{252}{100}$$
 b) $\frac{227}{90}$ c) $\frac{250}{99}$ d) $\frac{252}{99}$ e) NOTA

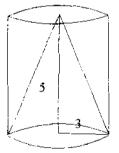
23. Factor completely:
$$x^4 + x^3z + x^3y + 3x^3 + 3x^2z + 3x^2y + 3x^2 + 3xz + 3xy + x + y + z$$
.

a)
$$(x+1)^{2}(x+z)$$
 b) $(x+y)^{2}(x+z)$ c) $(x+y+z)(x+1)^{2}$ d) $(x+y+z)(x+1)^{3}$ e)NOTA

24. Find:
$$(x - y)$$
, given: $\begin{bmatrix} x + y & 6 \\ 5 & 5 - x \end{bmatrix} + \begin{bmatrix} x - 2y & 2 \\ x & y \end{bmatrix} = \begin{bmatrix} 1 & z \\ y + 4 & 2y \end{bmatrix}$.
a) -1 b) 8 c) 5 d) 1 e) NOTA

25. Avi is going to a debate tournament. For the first 15 minutes he travels at a rate of 40 miles per hour. For the next half hour, he travels at a rate of 60 miles per hour and reaches the tournament. Immediately he decides to quit debate and go to his orchestra rehearsal 60 miles away. At what speed (miles per hour) must he now travel to ensure that the average speed for his entire trip is 50 miles per hour?

TB1) Find the volume of a shape, formed by removing a cone with radius 3 and height 5 from a cylinder with the same radius and height.



TB2) Find the sum of the distinct prime factors of 1224.

TB3) Find the sum of the greatest common factor and the least common multiple of 24, 57, and 84.