Algebra I Exam
Vestavia Hills High School Math Tournament
2014

1. Find the number of diagonals in a regular dodecagon.

   A. 12 B. 60 C. 36 D. 54 E. NOTA

2. Evaluate $5+\cfrac{2}{5+\cfrac{2}{5+\cfrac{2}{5+\ldots}}}$.

   A. $\frac{5+\sqrt{13}}{2}$ B. $\frac{5+\sqrt{33}}{2}$ C. $\frac{5+\sqrt{29}}{2}$ D. 5.3 E. NOTA

3. Find the area of an isosceles triangle with legs of length 13 and base of length 10.

   A. 30 B. 60 C. 90 D. 120 E. NOTA

4. Compute $75^3 - 45^3$.

   A. 30 B. 360 C. 900 D. 3600 E. NOTA

5. Mr. Taylor has a 7 ft by 10 ft rectangular doghouse. He wants to leave his dog, Barkley, in the yard while he goes to work. If Barkley's leash is 9 feet long and the leash is attached at an outside corner of the doghouse, what is the area of the space in which Barkley can roam? All answers are in square feet.

   A. $\frac{255}{4}\pi$ B. $\frac{137}{4}\pi$ C. $\frac{241}{4}\pi$ D. $\frac{247}{4}\pi$ E. NOTA

6. What is the probability that someone draws a face card and then a black card from a standard deck of cards, if the cards are drawn without replacement? A face card is a jack, queen, king, or ace.

   A. $\frac{2}{13}$ B. $\frac{4}{5}$ C. $\frac{8}{51}$ D. $\frac{4}{13}$ E. NOTA

7. Find the sum of the series $1+4+7+\ldots+34$.

   A. 210 B. 200 C. 247 D. 175 E. NOTA

8. A bee and a mosquito are 40 miles apart and begin flying toward each other. The fly travels at 3 mph and the mosquito travels at 5 mph. If a dog runs back and forth between the bee and mosquito at a speed of 7 mph, how far will the dog run until the bee and mosquito meet?

   A. 280 miles B. 74 miles C. 70 miles D. 35 miles E. NOTA

9. Simplify: $\frac{2\sqrt{3}+3\sqrt{2}}{3\sqrt{3}-2\sqrt{2}}$.

   A. $\frac{30+13\sqrt{6}}{19}$ B. $6\sqrt{3}+6\sqrt{2}$ C. $4\sqrt{6}$ D. $\frac{\sqrt{3}+\sqrt{2}}{5}$ E. NOTA

10. Greg can paint a house in 5 hours, and Jerry can paint a house of equal size in 12 hours. To the nearest tenth of an hour, how long would it take them to paint a house if they work together? All answers are in hours.

    A. 3 B. 3.5 C. 4 D. 4.5 E. NOTA

11. Every morning, a very slow mouse climbs a 5-foot pole to get some cheese that rests at the top of the pole. However, he can only make it up the pole 10 inches during the day, and he slides down the pole 6 inches each night. On which day does the mouse finally get the cheese?

    A. 14 B. 15 C. 16 D. 17 E. NOTA

12. The expression $\sqrt{\frac{32}{45}}+\sqrt{\frac{448}{147}}$ is equivalent to which of the following expressions?

A. $\frac{24\sqrt{6}+28\sqrt{35}}{21\sqrt{15}}$ B. $\frac{32\sqrt{14}}{21\sqrt{15}}$ C. $\frac{4\sqrt{2}+8\sqrt{7}}{21\sqrt{15}}$ D. $\frac{28\sqrt{6}+24\sqrt{35}}{21\sqrt{15}}$ E. NOTA

13. What is the left-most x-intercept of the graph of $y=x^2+4x-12$?

A. -12 B. 2 C. -6 D. -2 E. NOTA

14. The system of equations shown below can be solved with $x=A, y=B, z=C$. Find $A+B+C$.

$$\begin{cases} x+5y+3z=4 \\ 5x+2y+4z=9 \\ 2x+\ y+\ z=11 \end{cases}$$

A. 3 B. 4 C. 5 D. 2 E. NOTA

15. Find $\frac{f(x+h)-f(x)}{h}$ for $f(x)=(x+2)^2$.

A. $h+2x+4$ B. $h^2+2hx+4h$ C. $h+2x+2$ D. $2x+4$ E. NOTA

16. If one pound of gummy bears costs \$1.39 and a pound of gummy worms costs \$1.88, and you want to create a one-pound mixture of the two candies costing a total of \$1.60, how many ounces of gummy bears will you buy? Round to the nearest whole number.

A. 7 B. 9 C. 5 D. 6 E. NOTA

17. A 50 meter by 90 meter rectangular garden has a uniform rectangular sidewalk around it. If the area covered by the garden and sidewalk is 6000 m^2, how wide is the sidewalk, in meters?

A. 2.5 B. 7.5 C. 10 D. 5 E. NOTA

18. Bruster's has 20 flavors of ice cream. Jennifer wants one cone of three different flavors. How many different combinations of ice cream could she get?

A. 1140 B. 20! C. 6840 D. 5740 E. NOTA

19. Let r and s be the roots of the equation $8x^2-54x-45=0$. Find $\frac{r+s}{rs}$.

A. $\frac{27}{4}$ B. $\frac{5}{6}$ C. $-\frac{6}{5}$ D. 1 E. NOTA

20. If $f(x)=3x^2-12x-10$, find the value of $f(f(f(5)))$.

A. -55 B. 5 C. 55 D. -5 E. NOTA

21. For $f(x)=x^3-12x^2+8x-6$, let a represent the sum of the roots, b represent the sum of the product of the roots taken two at a time, and c represent the product of the roots. Find the value of $a+b+c$.

A. -10 B. 10 C. 26 D. -26 E. NOTA

22. Find all elements of $M=\left\{11,\sqrt{5},-10,0,\frac{0}{8},\sqrt{16},0.98\right\}$ that are rational numbers.

A. $11,0,\sqrt{16}$ B. $11,-10,0,\sqrt{16},0.98,\frac{0}{8}$ C. $\sqrt{5},\frac{0}{8},0.98$ D. $\sqrt{5},\sqrt{16}$ E. NOTA

23. Find the length of the segment of the line $y=\frac{1}{2}x+4$ in the interior of the parabola $y=x^2-\frac{9}{2}x+9$.

A. $\frac{5+\sqrt{5}}{4}$ B. $\frac{21-\sqrt{5}}{2}$ C. $\frac{8}{5}$ D. $\frac{5}{2}$ E. NOTA

24. If $64^{x+3}=4^{2x-1}$ and $216^{y-2}=36^{3y-1}$, find the value of $(x-y)^2-3xy$.

A. $\frac{796}{9}$ B. $\frac{1516}{9}$ C. $\frac{316}{9}$ D. $\frac{1036}{9}$ E. NOTA

25. Find the remainder when $x^2-9x+10$ is divided by $x+2$.

A. 8 B. 0 C. −4 D. 7 E. NOTA

Write the answers to the tie-breakers on the back of your bubble form. Denote each answer as T1, T2, and T3.

T1. Evaluate $\dfrac{-4(-6)-(3)(2)^3}{-12-\sqrt{144}+5}$.

T2. Solve for c: $\dfrac{1}{a}+\dfrac{1}{b}=\dfrac{1}{c}$.

T3. Factor over the integers: $6a^3+15a^2b-4ab^2-10b^3$.

You may keep your copy of the exam.