2011 Vestavia Hills High School
Mathematics Tournament
Algebra II Written Examination

1. How many unique numbers not beginning with 0 can be formed by rearranging the digits of 1,223,330?

A. 420 B. 7 C. 108 D. 360 E. NOTA

2. Find the distance between the directrices of $4x^2+8x-3y^2+6y-11=0$.

A. 3 B. $\frac{16}{5}$ C. $\frac{8}{3}$ D. 4 E. NOTA

3. The number $(2^{48}-1)$ is evenly divisible by two numbers between 60 and 70. Find the sum of the two numbers.

A. 124 B. 126 C. 128 D. 130 E. NOTA

4. The solution set to $\frac{x-4}{x+5}+\frac{x-1}{x+3}<2$ is $(a,b)\cup(c,\infty)$. Find $[a+b]+c$, where [ ] represents the greatest integer function.

A. −12 B. 9 C. −11 D. −3 E. NOTA

5. A multiple choice test contains five questions with four possible answers for each question. Each question has only one correct answer. What is the probability that a student who knows nothing about the test material will answer at least three of the questions correctly?

A. $\frac{1}{4}$ B. $\frac{21}{1024}$ C. $\frac{53}{512}$ D. $\frac{3}{256}$ E. NOTA

6. If $A=\left|\begin{bmatrix} 9 & -3 \\ 2 & 9 \end{bmatrix}\times\begin{bmatrix} 4 & 7 \\ -6 & 8 \end{bmatrix}\right|$, find $\frac{A}{2}$.

A. 2850 B. 6438 C. 4644 D. 1425 E. NOTA

7. Find the coefficient of the fourth term in the expansion of $(x+2y)^{\frac{10}{3}}$.

A. $\frac{280}{243}$ B. 15360 C. $\frac{140}{81\sqrt[3]{2}}$ D. $\frac{1120}{81}$ E. NOTA

8. On every birthday of his life, Fidelio has put as many pennies in a piggy bank as his age in years. If he now has $1.20 in the jar, what is Fidelio's age?

A. 12 B. 15 C. 16 D. 18 E. NOTA

9. The number of solutions of $2^{2x}-3^{2y}=55$ in which x and y are positive integers, is

A. Zero B. One C. Two D. Three E. NOTA

10. If $\sum_{n=0}^{\infty}\cos^{2n}\theta=4$, find $\cos 2\theta$.

A. $\frac{\sqrt{3}}{3}$ B. $\frac{1}{4}$ C. $\frac{\sqrt{3}}{2}$ D. $\frac{1}{2}$ E. NOTA

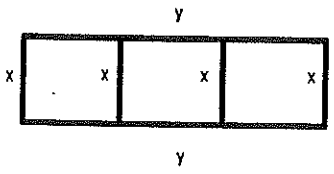
11. Line l is tangent to the circle $x^2+y^2=2011$ at $P(a,b)$ in the 1st quadrant. If the intercepts of l and the origin form the vertices of a triangle whose area is 2011, compute $a\cdot b$.

A. $\frac{2011}{2}$ B. 2011 C. $\frac{2011}{4}$ D. 4022 E. NOTA

12. Find the number of digits in 5^{2011}.

A. 1006 B. 1405 C. 1005 D. 1406 E. NOTA

13. Mr. Taylor wants to build a rectangular pen for his three dogs, and he wants each pen to have the largest possible area for his dogs to roam. If he has 300 feet of fencing available, and if x and y represent the values of these dimensions, find $\frac{x^2}{y^2}$.



A. 1 B. $\frac{9}{16}$ C. $\frac{1}{9}$ D. 9 E. NOTA

14. Find the smallest positive integer that satisfies $\frac{\log_3(4x)-\log_3 5+3}{6}>1$.

A. 31 B. 33 C. 35 D. 37 E. NOTA

15. Find the sum of the infinite series: $1-\frac{1}{3}-\frac{1}{5}+\frac{1}{8}+\frac{1}{9}-\frac{1}{25}+\frac{1}{64}\cdots$

A. $\frac{8}{7}$ B. $\frac{8}{15}$ C. $\frac{10729}{14400}$ D. $\frac{9}{14}$ E. NOTA

16. Let A = the volume of a cube with side length 9

B = the lateral area of a cylinder with radius $\frac{7}{2}$ and height 10

C = the sum of the squares of the first 10 natural numbers

D = the number of real roots of the polynomial $150x^9+10x^6-5x^5+4x^4-2=0$

E = the dot product of two orthogonal vectors

Find $ADE+\frac{B}{C}$.

A. $\frac{\pi}{11}$ B. $\frac{\pi}{7}$ C. $\frac{2\pi}{11}$ D. $\frac{2\pi}{7}$ E. NOTA

17. Which of the following is the identity function $f(x)=x$ for all real numbers?

A. $f(x)=e^{\ln x}$ B. $f(x)=\ln e^x$ C. $f(x)=\sin\left(\sin^{-1}x\right)$

D. $f(x)=\tan^{-1}(\tan x)$ E. $f(x)=\sqrt{x^2}$

18. If (x,y) is the hole in the graph of $y=\frac{x^2+2058x+94517}{x+2011}$, find $|x|-|y|$.

A. 47 B. 2012 C. 2011 D. 2010 E. NOTA

19. Let $S = 2+4+6+...+2N$, where N is the smallest positive integer such that $S > 1{,}000{,}000$. Find the sum of the digits of N.

A. 27 B. 12 C. 6 D. 2 E. NOTA

20. If an item is sold for x dollars, there is a loss of 15% based on the cost. If, however, the same item is sold for y dollars, there is a profit of 15% based on the cost. Find ratio of $y:x$.

A. 30:7 B. 23:17 C. 21:4 D. 17:23 E. NOTA

21. Find the equation whose roots are 2 greater than the roots of the equation $3x^3 - 2x^2 - 5x + 2 = 0$.

A. $3x^3 + 16x^2 + 23x + 8 = 0$ B. $3x^3 - 20x^2 + 39x - 20 = 0$ C. $3x^3 - 4x^2 - 20x + 16 = 0$
D. $3x^3 - 4x^2 - 7x + 4 = 0$ E. NOTA

22. When simplified, the third term in the expansion of $\left(\frac{a}{\sqrt{x}} - \frac{\sqrt{x}}{a^2}\right)^6$ is

A. $\frac{15}{x}$ B. $-\frac{6x^2}{a^9}$ C. $-\frac{15}{x}$ D. $\frac{20}{a^3}$ E. NOTA

23. If $f(x) = px + q$ and $f(f(f(x))) = 8x + 21$, and if p and q are real numbers, find the value of $p + q$.

A. 2 B. 3 C. 5 D. 7 E. NOTA

24. For how many values of the coefficient a does the system $\begin{cases} x^2 + ax + 1 = 0 \\ x^2 - x - a = 0 \end{cases}$ have a common real solution?

A. 0 B. 1 C. 2 D. infinitely many E. NOTA

25. Five holes of increasing size are cut along the edge of one face of a box. The number of points scored when a marble is rolled through that hole is the number above the hole. There are three sizes of marbles. The smallest marbles will fit through any of the holes; the medium marbles will only go through holes 3, 4, and 5; the largest only through 5. You may choose up to 10 marbles of each size to roll and every rolled marble goes through a hole. For a score of 23, what is the maximum number of marbles that could have been rolled?

A.12 B. 13 C. 14 D. 15 E. NOTA

PLEASE WRITE YOUR NAME, COMPLETE SCHOOL NAME, AND THE TIE-BREAKER ANSWERS ON THE BACK OF THE SCANTRON FORM. DENOTE EACH TIE-BREAKER AS T1, T2, AND T3.

T1: Let $f(x) = \frac{\sqrt{x^2 - 1}}{x}$. Find the set of all real values of x for which $f(f(x))$ exists.

T2: $\left\| \begin{bmatrix} 11 & 0 & 6 & 5 \\ 1 & 3 & 7 & 8 \\ 2 & 13 & 3 & 0 \\ 9 & 4 & 1 & 4 \end{bmatrix} \right\|$

T3: Find the sum of the infinite series: $\frac{10}{11} + \frac{13}{121} + \frac{16}{1331} + \frac{19}{14641} \cdots$

YOU MAY KEEP THIS COPY OF THE EXAM.