

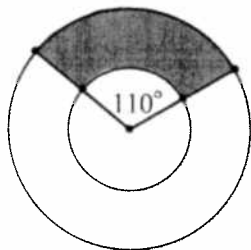
**Vestavia Hills High School Mathematics Tournament 2011**  
**Geometry Ciphering**

Practice: Find the area of a regular octagon with side length 2.

Ans:  $8 + 8\sqrt{2}$

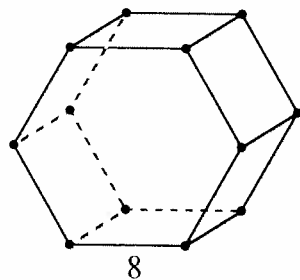
1.1 The circles are concentric. The smaller circle has radius 3 and the larger circle has radius 5. Find the area of the shaded region.

Ans:  $\frac{44}{9}\pi$



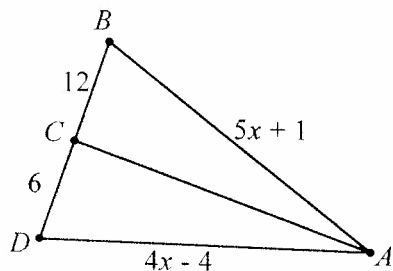
1.2 The volume of the right hexagonal prism is  $384\sqrt{3}$ . If each base edge is 8, find the surface area of the prism.

Ans:  $192 + 192\sqrt{3}$



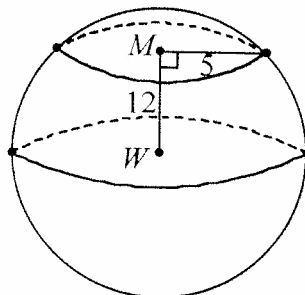
1.3  $\overline{AC}$  bisects  $\angle BAD$ . Find the perimeter of  $\triangle ABD$ .

Ans: 42



1.4 Circles  $M$  and  $W$  are parallel. The distance between the great circle  $W$  and circle  $M$  is 12 units. The radius of circle  $M$  is 5 units. Find the volume of the sphere.

Ans:  $\frac{8788}{3}\pi$

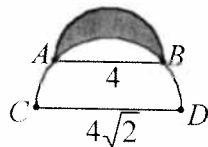


1.5 A triangle with vertices of  $(0, 3)$ ,  $(3, 5)$ , and  $(0, 10)$  is rotated around the  $y$ -axis. What is the volume of the resulting figure?

Ans:  $21\pi$

2.1  $\overline{AB}$  is a diameter of the smaller semicircle and  $AB = 4$ .  $\overline{CD}$  is the diameter of the larger semicircle and  $CD = 4\sqrt{2}$ . Find the area of the shaded region.

Ans: 4

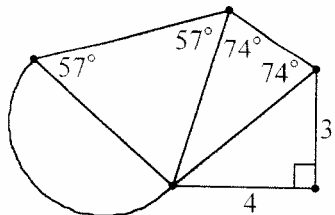


2.2 Andrea paints a  $4 \times 4 \times 4$  wooden cube chartreuse and Wendi cuts it into 64 unit cubes with her awesome ninja skills. How many of them have 2 or more sides painted?

Ans: 32

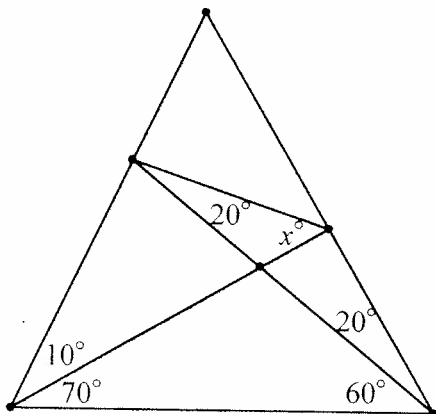
2.3 Find the length of the radius of the semicircle.

Ans:  $\frac{5}{2}$



2.4 Solve for  $x$ .

Ans:  $110^\circ$



2.5 Find the area of a triangle with vertices  $(1, 1)$ ,  $(2, 4)$  and  $(6, 3)$ .

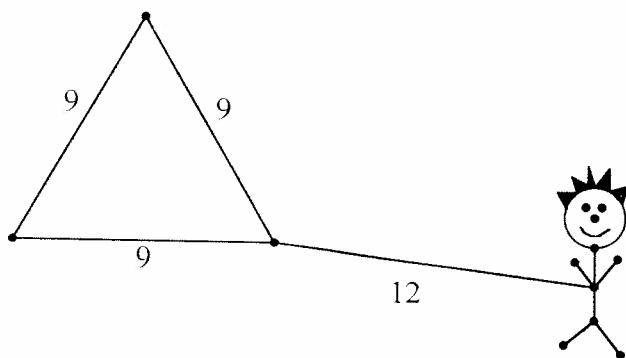
Ans:  $\frac{13}{2}$

3.1 Find the dimensions of all rectangles with integer side lengths such that the area is numerically two times the perimeter.

$5 \times 20$   
Ans:  $6 \times 12$   
 $8 \times 8$

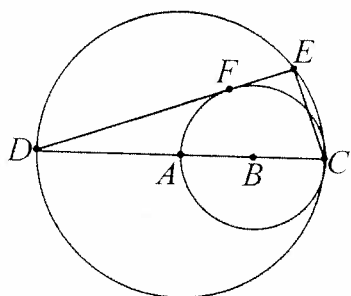
3.2 Find the area in which Mr. Taylor can roam.

Ans:  $126\pi$



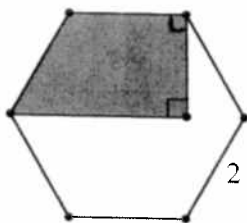
3.3 In the diagram,  $\overline{DC}$  is a diameter of the larger circle centered at  $A$ , and  $\overline{AC}$  is a diameter of the smaller circle centered at  $B$ . If  $\overline{DE}$  is tangent to the smaller circle at  $F$ , and  $DC = 12$ , find  $DE$ .

Ans:  $8\sqrt{2}$



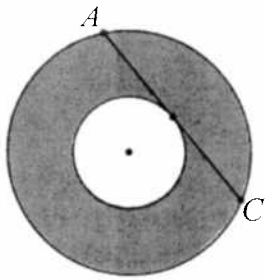
3.4 The hexagon in the diagram is regular with side length 2. Find the area of the shaded region.

Ans:  $\frac{5}{2}\sqrt{3}$



3.5 Find the area of the annulus if  $AC = 2048$  and the chord  $\overline{AC}$  is tangent to the smaller circle.

Ans:  $1048576 \pi$

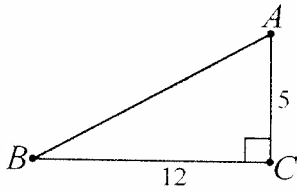


4.1 Assume that a hot air balloon is in the form of a perfect sphere. The balloon is rising into the sky at a rate of 502 meters per second. Assuming that gravitational pull creates an acceleration of  $-9.8$  meters per second and that the  $x$  velocity is zero, find the volume of the air inside the balloon if the diameter is 6 m

Ans:  $36\pi \text{ m}^3$

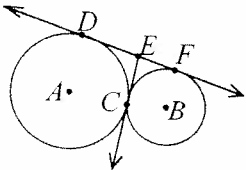
4.2 Find  $\sin A + \sin B - \cos A - \cos B$

Ans: 0



4.3  $CE = 3$ . Find the length of the common external tangent  $\overline{DF}$

Ans: 6

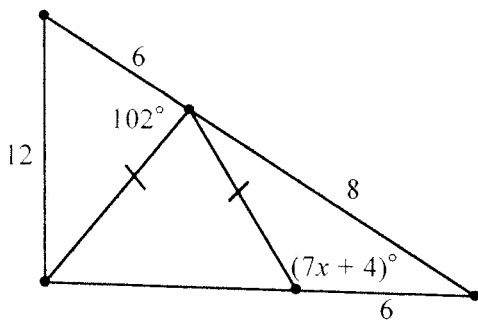


4.4 Find the coordinates of the centroid of a triangle whose vertices are  $(24, 6)$ ,  $(5, 13)$ , and  $(1, 8)$ .

Ans:  $(10, 9)$

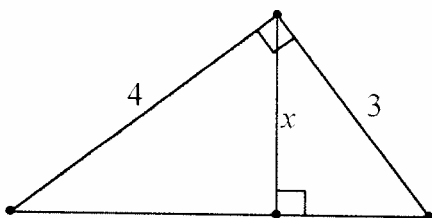
4.5 Write an inequality or pair of inequalities to describe the possible values of  $x$ .

Ans:  $-\frac{4}{7} < x < 14$



E1: Find  $x$ .

Ans:  $\frac{12}{5}$



E2: If possible find the value(s) of the real common solution of  $y$  for the pair of equations

Ans: 4

$$x^2 + y^2 - 16 = 0$$

$$x^2 - 3y + 12 = 0$$

E3: Find  $x$ .

Ans:  $\sqrt{73 - 24\sqrt{3}}$

