# 2006 Hoover HS Math Tournament Algebra II Ciphering

Practice Find the sum of all values of $x$ for which $x+2=(x+1)^2$ -1

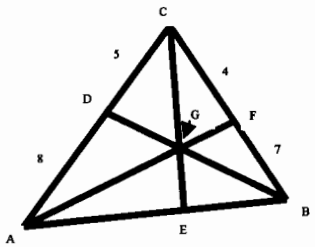
1.1 If $y = 4z$ is a complex number such that $y^4 = -3+4i$, find $|z|$. $\frac{\sqrt[4]{5}}{4}$

1.2 Solve for $x$: $\log(|x+5|) - \log(|x+8|)=1$ $-\frac{75}{9}$

1.3 Solve for $y$: $\left(\sqrt[2]{2}y\right)\left(\sqrt[3]{2}y\right)\left(\sqrt[6]{2}y\right)=1458$ 9

1.4 Find the smallest positive integer $x$ such that $f(f(x))=x$ for $f(x)=\frac{x+1}{x-1}$ 2

1.5 Find the ratio BG:GD in the diagram $\frac{91}{32}$




2.1 Find the sum: $\sum_{n=1}^{20}(2n^2-3)$ 5680

2.2 Find the vertex that lies in the second quadrant of the conic section with equation $4x^2-3y^2+8x=-16$ $(-1,2)$

2.3 Buzz takes a trip to the North Pole. He averages 25 miles per hour going to the North Pole and 10 miles per hour coming back. What is his average speed for the entire trip, in miles per hour? $\frac{100}{7}$

2.4 Factor into irreducible polynomials: $f(x)=x^4-x^3-x+1$ $(x-1)^2\left(x^2+x+1\right)$

2.5 Find the sum: $\sum_{n=1}^{\infty}\frac{1}{n(n+4)}$ $\frac{25}{48}$

3.1 If $\sin\alpha=\frac{2}{3}$ and $\cos\beta=\frac{1}{5}$ with $\alpha$ and $\beta$ both in the first quadrant, find $\cos(\alpha+\beta)$. $\frac{\sqrt{5}-4\sqrt{6}}{15}$

3.2 Write in $a+bi$ form: $\left(\frac{1-i}{1+i}\right)^{2006}$ -1

3.3 Find the determinant of the matrix: $\begin{bmatrix} 1 & -1 & 3 & 4 \\ 2 & 0 & -2 & -2 \\ -2 & 0 & 2 & -2 \\ -1 & 1 & 3 & -4 \end{bmatrix}$ 48

3.4 How many of the seven complex seventh-roots of $7i$ lie in the third quadrant of the complex plane? 1

3.5 The sum of the cubes of two positive integers is 407. The cube of the sum of those two positive integers is 1331. Find the greater of the two integers. 7

4.1 The equation $y=a(b)^x$, where $a$ and $b$ are both positive, goes through the points (2,0.25) and (-2, 16). Find the value of $\frac{1}{ab}$. $\sqrt{2}$

4.2 Find the largest value of $\log_x 3$ that satisfies the equation $\log_x 3 + \log_3 x = 3$ $\frac{3+\sqrt{5}}{2}$

4.3 Find the eccentricity of the conic section given by the equation $3x^2 - 7y^2 + 12x - 14y - 16 = 0$ $\frac{\sqrt{70}}{7}$

4.4 A sequence is defined by $a_1 = 3$ and $a_{n+1} = \frac{a_n + 7}{2}$. What is the smallest number that is larger than every term in this sequence? 7

4.5 Find the coefficient of the term containing $x^{-\frac{1}{2}}$ in the expansion of $(x-1)^{\frac{1}{2}}$ $-\frac{1}{2}$

E.1 Solve the inequality, and write your answer in interval notation: $\frac{x^3 - 1}{x - 1} \geq 0$ $(-\infty, 1) \cup (1, \infty)$

E.2 Find the largest integer value of the function $f(x) = -2x^2 - 3x + 3$ 4